


1972

# Using computer assisted instruction in overcoming attitude barriers

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Using computer assisted instruction  
in overcoming attitude barriers

by

Lois Haase Kockler

A Dissertation Submitted to the  
Graduate Faculty in Partial Fulfillment of  
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## INTRODUCTION

The use of computers as instructional aids in our educational institutions has become more common in recent years, to such an extent that "direct use of computers by teachers and students throughout the country is now an actuality" (99, p. 90). Since the area of mathematics is particularly well-suited to this mode of instruction, much of the early computer-assisted instruction (CAI) work was done in this field. On the primary level, computer programs that give the individual child practice with arithmetic problems appropriate for his own level have already been developed and tested with favorable results (50, 87, 95), and at higher educational levels computers have been used in many capacities to provide instruction in such diverse areas as foreign languages, mathematics and statistics, sciences, and social sciences (41). Furthermore, it seems likely that technological development will permit and encourage increased usage of computers as instructional aids in schools and colleges within the foreseeable future.

## The Problem

Despite the expanded use of CAI and the probability of further expansion, there has been very little research in CAI that deals with the affective as well as the cognitive domain. This investigation, however, began with the idea that the quality of teaching ought to be judged not only on the amount of information the student has digested or on the skills he has gained, but on the student's



attitudes as well. That is, if a student is unhappy with his classroom situation in mathematics, he may be discouraged from continuing his study in that field, or he may develop a dislike for mathematics or a fear of mathematics that causes him to avoid using the knowledge he has gained. When the students themselves are prospective primary teachers, their attitude toward mathematics assumes particular importance, since these students may transmit a negative attitude to another generation of scholars. One researcher summed up the problem by saying that in mathematics, "the results of research have suggested that the teacher, perhaps even more than the parents, is an important determiner of student attitudes" (97, p. 40).

Furthermore, the attitudes of prospective primary teachers toward CAI would also be of special interest, since these students may have the opportunity to use the computer within their own classrooms at some time in their career. Yet Bishop (9), who researched programs of teacher education institutions in Missouri and adjoining states, concludes that there is currently very little instruction in CAI for future teachers, and a search of college catalogs indicates that this conclusion is probably valid in other regions as well. Unfortunately, if little instruction is being provided in the use of CAI, it is unlikely that these teachers will be psychologically and technologically prepared to integrate this media into their instructional strategies. Tobias (89) found among teachers a bias against terms describing newer forms of instructional media, causing him to advocate instruction in these media for teacher

trainees. It seems reasonable to this investigator that such biases against CAI might be removed through the actual use of the computer in a student's academic program. Ellison states that teachers need instruction in CAI through their methods courses, but he insists that "first, the teacher needs to gain part of his own education through the medium of the computer" (25, p. 1), and a report sponsored by the Organisation for Economic Cooperation and Development states the following warning (70, p. 12):

Frequently in the past, innovations have been brought into education at the student level before teachers have had an opportunity to experience their effect at first hand. This does not seem to be a desirable approach in terms of the accomplishment of change in a smooth and efficient way. It would seem more appropriate to introduce computing activities into teacher training, and in-service training of teachers, as soon as possible in such a way as to encourage a multiplying effect throughout the education system and to make their utilisation with students more efficient and widespread.

#### Purpose of the Study

Assuming that gains in mathematics achievement, gains in attitude toward mathematics, and gains in attitude toward CAI are worthy goals, particularly for prospective primary teachers, it is reasonable to ask whether the use of CAI as a part of a student's academic work will accomplish any or all of these goals. Thus the central objectives of this study are as follows:

1. To develop six CAI units. Each unit is an automated programmed instruction lesson that covers a topic normally included in math 190, a course designed primarily for elementary education majors, at Iowa State University. For the experimental group (the

CAI group), these six units will be used as a replacement for six traditional classroom lectures covering the same topics.

2. To determine if the use of CAI as a part of an undergraduate mathematics course can change the student's attitude toward CAI.

3. To determine if the use of CAI as a part of an undergraduate mathematics course can change the student's attitude toward mathematics.

4. To compare gains of knowledge of mathematics made by CAI students with gains made by students in a conventional lecture situation (the control group).

Secondary objectives of the study are the following:

5. To investigate the possible relationship among the three components in the study (attitude toward CAI, attitude toward mathematics, knowledge of mathematics).

6. To determine the amount of change in student attitude that occurs after each CAI lesson.

The following null hypotheses were stated for investigation:

1. There is no significant change in the control group's attitude toward CAI as a result of taking math 190.

2. There is no significant change in the CAI group's attitude toward CAI as a result of taking math 190. For both the control group and the CAI group, attitude toward CAI is measured by pre-test and post-test administrations of a questionnaire developed by Brown in 1966 (68).

3. There is no significant difference in the control group's attitude toward CAI and the CAI group's attitude toward CAI (as measured by the post-test) when initial pre-test differences have been controlled.

4. There is no significant change in the control group's attitude toward mathematics as a result of taking math 190.

5. There is no significant change in the CAI group's attitude toward mathematics as a result of taking math 190. For both the control group and the CAI group, attitude toward mathematics is measured by pre-test and post-test administrations of a questionnaire developed by Aiken and Dreger in Aiken (1, p. 477).

6. There is no significant difference in the control group's attitude toward mathematics and the CAI group's attitude toward mathematics (as measured by the post-test) when initial pre-test differences have been controlled.

7. There is no significant change in the control group's achievement in mathematics as a result of taking math 190.

8. There is no significant change in the CAI group's achievement in mathematics as a result of taking math 190. For both the control group and the CAI group, achievement in mathematics is measured by pre-test and post-test administrations of a 24-item test constructed by the investigator.

9. There is no significant difference in the control group's achievement in mathematics and the CAI group's achievement in mathematics (as measured by the post-test) when initial pre-test

differences have been controlled.

Additionally, the following steps were taken to fulfill the secondary objectives of the study:

1. The correlation coefficient between each pair of component variables (pre-test attitude toward CAI, pre-test attitude toward math, pre-test achievement in math, post-test attitude toward CAI, post-test attitude toward math, and post-test achievement in math) was calculated.

2. A randomly-selected half of the CAI group took a seven-item questionnaire after completing each CAI lesson. The questionnaire, which was designed as a short measure of attitude toward CAI, was administered to a subsample of the CAI group to avoid interference with the main questionnaire on attitudes toward CAI.

#### Sample for the Study

The sample for the study was drawn from 243 students registered for math 190 (mathematical concepts) at Iowa State University during winter quarter of 1972. Although the enrollment did include some junior and senior undergraduates, most of the students were freshmen or sophomores, and almost all students were enrolled in the class to fulfill their mathematics group requirements. Thirty-two of the 243 students were majoring in elementary education; of the remaining students, most were majoring in fields such as child development, music, home economics, physical education, or agriculture.

The textbook for the course was Elementary Mathematics by Rex Hutton, published by International Textbook Company in 1971. For math 190, the course content consists of the first six chapters of this text supplemented by additional material which the individual instructor believes to be appropriate. The topics chosen for the CAI units are the following: the base twelve numeration system, including numerals with a radial point; the properties of subtraction and the commutative property of arbitrary operations; a definition of division and properties of division; the tests for divisibility by 2, 3, 4, 5, 6, 8, 9 and 10; the definition of prime numbers and a method for determining whether a number is prime; and the definition and three methods of finding the least common multiple of two natural numbers.

#### Organization of the Study

The material included in this study was organized in the following six chapters: Introduction, Review of Literature, Methods and Procedures, Findings, Discussion, and Summary. The first chapter contains a statement of the problem, purposes of the study, sample for the study, and organization of the study. In chapter two the literature review includes an overview of CAI and a review of studies in the following categories: studies dealing with cognitive gains through tutorial CAI; with cognitive gains through problem-solving; with cognitive gains through evaluation; with attitudes toward CAI; and with attitude changes through the use of

CAI. The methods chapter, chapter three, deals with the development of the CAI units and other materials for the study, the experimental procedure, and the statistical treatment of the data. Chapter four presents the findings of the data, and chapter five provides a discussion of these findings and recommendations that stem from the findings. A summary of the project and the investigator's conclusions from the research are stated in chapter six.

## REVIEW OF LITERATURE

A review of the literature of computer-assisted instruction (CAI) is complicated somewhat by the fact that the term "CAI" is used to describe a variety of computer uses in the educational process. Hansen defines CAI as "a form of human machine interaction whose goal is the efficient learning of the desired curriculum" (37, p. 3), and he lists five types of CAI with the following descriptions (37, p. 2):

- (1) drill and practice that provides a potential automation of the problem-solving routines or homework to be mastered by a student;
- (2) tutorial approaches that attempt to replace the teacher in as complete a manner as possible;
- (3) problem-solving tasks that use the computer both as a problem-structuring device and as a calculational tool for generating answers;
- (4) simulation problems that attempt to replace many of the empirical activities such as found in our science laboratory with symbolic representations provided by the logical and stochastic capabilities of computers; and
- (5) evaluation tasks via computer that result both in sequential testing and more sophisticated forms of data analysis.

Another major researcher, Zinn (98), omits the "evaluation" category in his list, but adds the categories "retrieval and reorganization of information" and "artistic design and composition." Others use a much more restricted definition of CAI, as does Harless et al. when they define CAI as "a form of programmed instruction where the computer simulates an individual tutorial and is programmed to present information employing various pedagogical strategies" (38, p. 86). This definition would include only category four, tutorial CAI, under Hansen's definition.

Throughout this paper, the term "CAI" will be used in the sense



of Hansen's definition, so that the full range of student-computer interaction may be included. In the author's judgment, however, CAI research conducted with elementary school children and most CAI research in non-technical subjects falls beyond the scope of this paper, so research in these two categories is generally not included in this review. Research that is included may be classified as follows: studies concerned with cognitive gains through tutorial CAI; studies concerned with cognitive gains through problem-solving; a study concerned with cognitive gains through evaluation; studies in attitudes toward CAI; and studies concerned with the effects of CAI on attitudes toward mathematics.

#### Cognitive Gains Through Tutorial CAI

In studies employing tutorial CAI, as in countless other educational studies, the results have frequently shown no significant differences in achievement between the CAI group and the control group, although there is often "a considerable savings of time in favor of CAI" (35, p. 7). One such study is that of Longo (58) in which 278 students in basic electronics were given a week of CAI and a week of traditional instruction, while an equal number of students were given two weeks of traditional instruction in the same subject. There were no significant differences in achievement between the two groups, but the mean time spent by the CAI group during their week on the computer was 8.99 hours, compared to 11.25 hours of instruction for the control group.

There was also a time difference favoring CAI in a study by Wassertheil (94) in the field of statistics. In this research, 14 of the 27 students in an elementary statistics class spent sixty minutes a week in CAI instead of attending the seventy-five-minute problem lab that the control group attended each week. Using the course final exam as the criterion, the CAI group performed better than the control, but not significantly better. It should be pointed out, however, that the students in the CAI group were all volunteers, and authorities such as Borg warn that "samples of volunteers can be assumed to be biased, and the results of studies employing volunteers must usually be discounted because of this bias" (12, p. 176).

Other experiments in tutorial CAI that showed no significant differences between the CAI group and the control group are those of Cole (16), Riedesel and Suydam (75), Brown and Gilman (14), and Roid (77). The study by Cole used CAI to improve basic math skills of students in a senior high math skills class. The students did show a significant gain in computational skills, but there was no significant difference between their scores and the scores of a traditionally-instructed control group.

In a college level course, Riedesel and Suydam (75) used CAI to teach mathematics to prospective primary teachers, and compared the achievement of this group with the achievement of a control group that attended classroom lectures. Again, the CAI group and the control group did not differ significantly in an achievement

test administered at the end of the term. In this study, time spent in instruction was roughly equivalent for both groups.

Brown and Gilman (14) conducted a study with two CAI groups that differed in the kind of feedback they received, and a control group that studied a programmed text. All groups studied the same topics in high school physics, but both CAI groups required more time to complete the instruction than did the textbook programmed instruction group. The three groups showed no significant differences in scores on either the immediate post-test or on a six-week retention test.

Roid's study (77) also compared the achievement of a CAI group to the achievement of a group using printed programmed instruction, but in this case the subject studied was psychology and the treatment consisted of three lessons of thirty-five frames each. The CAI group improved significantly between pre-test and post-test administrations of an exam on each of the three units, whereas the printed programmed instruction group showed significant improvement on only two of the three unit exams. When lesson pre-test scores were used as covariates, however, an analysis of covariance revealed no significant differences between the two groups. In this study, the programming was a frame-by-frame program with no branching to provide for individualization of instruction; hence one of the advantages of the computer, its ability to use a student's past performance in selecting later questions, was not utilized.

A study conducted at the State University of New York, Buffalo,

(27) attempted to compare CAI, textbook programmed instruction, and laboratory sessions conducted by a graduate assistant as modes of supplementing a graduate course in educational statistics, but the lack of controls on this experiment make the results highly questionable. Twenty students were assigned to attend the weekly laboratory sessions, but fifteen of these failed to attend even one of the sessions. The students in the CAI group differed substantially on the number of lessons they completed, and several of the programmed text group failed to use the text at all. On the other hand, two people assigned to the lab sessions also used the programmed text, either in addition to attending the lab or as a replacement for attending the lab. When final course grade was used to measure achievement, there were no significant differences among the three groups.

Studies using tutorial CAI that revealed significant differences in achievement between the CAI group and the control group include those of Culp and Lagowski (19), Ford and Slough (29), Ibrahim (44), Dick and Latta (20), and Roy (78). Culp and Lagowski (19) report three studies that used CAI as a supplement to courses in college chemistry. In the first study, students who used the CAI lessons scored significantly better than the control group, which received no supplementary treatment, on four out of five measures of achievement covering materials in the CAI lessons. These students did not score higher on course material that was not presented in the CAI lessons, however. In the second study reviewed by Culp, the control group again received no supplemental

treatment. In this case, the CAI group scored higher on four of the six achievement measures administered throughout the course. The third study also used a control group that received no supplemental instruction and a CAI group, but this time a third group of students was used, and these students attended sessions conducted by a graduate assistant in addition to their usual classwork. On four of six achievement measures, the CAI group performed significantly better than either of the other two groups.

Ford and Slough (29) used CAI to teach a segment of a basic electronics course to 51 students, while 200 students received traditional instruction in the same topics. On the standard Army electronics exam, the CAI group scored significantly better than the control group.

In a study by Ibrahim (44), 80 calculus students were randomly divided into groups that would receive one of the following four treatments: 1) four hours of CAI lecture and two hours of problem-solving by CAI, 2) four hours of lecture by CAI and two hours of problem-solving by a teacher, 3) four hours of lecture by a teacher and two hours of problem-solving by a teacher, and 4) four hours of lecture and two hours of problem-solving by a different teacher. On an achievement test administered shortly after the treatment, students who received CAI only (group 1) performed significantly better than the other groups, but there were no significant differences among the groups on a retained achievement test.

The idea of significant figures was taught to one group of

eighth graders by CAI, and to another group of eighth graders by printed programmed instruction in a study by Dick and Latta (20). This experiment resulted in a significant difference in achievement favoring the printed programmed group, although the authors state that the difference was largely attributable to the very low performance of several low ability students who used CAI. The experiment utilized cathode ray terminals, which leave no printed record of the computer's statements or of the student's responses, and the authors suggest that low ability students may need printed memory aids to supplement this type of CAI.

Another study in which CAI students were out-performed was conducted by Roy (78) using graduate students who were taking a course in Boolean algebra and logic design. On a retention test, the performance of students who had received traditional instruction in the subject was "markedly superior" to the performance of the ten students who took the course by CAI. The author cautions against generalizing this finding, however, because the study was plagued by problems with the computer programs and problems with the machine itself. The author indicated that circumstances had forced him to start the experiment before the system had been adequately tested.

A final study in tutorial CAI is a study by Hall (35) that gave inservice training in mathematics to primary teachers. Because most of the teachers who participated in the program lived in comparatively remote areas, (the Appalachian region), it would

have been extremely difficult to recruit an adequate number of students for a traditional lecture class in mathematics; hence this experiment did not use a control group. For the 287 students who completed the course, the average amount of time spent working on the computer was 19 hours, not including testing periods. Achievement was measured by two forms of an eighty-item multiple choice test, and students completing the course achieved a mean of 52.60% correct on the pre-test, and a mean of 72.54% correct on the post-test.

#### Cognitive Gains Through Problem-solving

A different approach to CAI in mathematics is the utilization of the computer's problem-solving ability in the instructional process. In this mode, the computer is looked at as a computer rather than as a sophisticated device for presenting programmed instruction, and the student is usually required to gain some facility in a computer language such as FORTRAN or BASIC in order to use the computer. The student then uses the computer to perform calculations that would be prohibitive by hand, but that help the student understand a function, a statistical distribution, an integral, or another mathematical concept.

Examples of the problem-solving mode of CAI applied to topics in introductory calculus are experiments by Schmidt (80), Fielder (28), Holoiien (43), and Bitter and Slaichert (10). In each of these studies, students were randomly divided into a CAI group and

a control group. Students in the CAI group programmed the computer to investigate assigned topics in calculus instead of working the control group's more traditional homework problems on those same topics. A typical set of topics, those included in the study by Schmidt, are limits, extrema, functional evaluation, and integration.

Fielder (28) observed no significant differences in achievement between the CAI group and the control group, and Schmidt (80) found that the means on all five achievement tests were higher for the CAI group, although the mean was significantly higher on only one of the five tests. Bitter and Slaichert (10), who used a CAI class and a control class at each of three colleges, reported a significant difference favoring the CAI group in differential calculus, but not on the integral calculus topics. Using two classes, each containing a CAI group and a control group, Holoien (43) found no significant difference in achievement between the CAI group and the control group in one class, but in the second class, there was a significant difference favoring the CAI group.

Bell (8) completed an experiment similar to the calculus studies described in the preceding paragraphs except that the control group in his experiment also learned computer programming. The difference was that the experimental group applied this programming to solving calculus problems, whereas the control group did not. No significant differences were found in the student's mastery of the techniques of calculus, but there was a significant difference favoring the experimental group in questions testing the



understanding of concepts of calculus.

Also using the problem-solving approach are Kiernen (51) and Hatfield (39), although each of these researchers worked with students below the college level. Kiernen's subjects were high school students in intermediate mathematics, and his experiment was conducted on two classes with a CAI group and a control group in each class. Using the exams in each class as separate measures, the study included 20 criterion measures in all. Of these, there was one significant difference favoring the control group and two favoring the CAI group, but means on 17 of the 20 measures favored the CAI group.

Hatfield (39) conducted his experiment in a seventh grade math class for two consecutive years. During the first year, there was a significant difference in one of the six measures, and that difference favored the control group. The computer programs were revised before they were used again, however, and the second year resulted in two significant differences in the six areas, both favoring the CAI group.

#### Cognitive Gains Through Evaluation

A final experiment in CAI uses the evaluative approach. Rockhill (76) developed a CAI program to test college students enrolled in a pre-calculus course on topics judged necessary for success in calculus. The computer analyzed the student's difficulties, and for each student the computer provided a listing of

instructional materials to correct the difficulties. Achievement tests revealed no significant difference between students who used the computer and students who did not use the computer.

#### Attitudes Toward CAI

In a 1971 paper on the status of computers in the educational process, Singh and Morgan summarized attitudinal research by saying simply that "there is little information available regarding student attitudes toward CAI" (84, p. 10). The reason for this scarcity of information may be that most students evaluating CAI have experienced it only through one course, and for a short period of time. A student's evaluation would then be an evaluation of the specific instructional program he had experienced rather than an evaluation of CAI in general, so over-all conclusions about attitudes toward CAI may be difficult to obtain. Furthermore, a student's attitude toward CAI may well be colored by his attitude toward the subject he has studied through CAI. Brown and Gilman consider this possibility in the following statement (14, p. 41):

It seems likely that student attitude is at least partially a function of the specific characteristics of the student-subject matter interface. If this is the case, various research findings may be to some extent system specific.

On the other hand, there may be little information available on attitudes toward CAI simply because most researchers have chosen to investigate the value of CAI in improving student achievement, since student attitudes toward CAI will be of no importance unless

CAI actually is effective in improving student achievement. In formulating a research project, therefore, most investigators choose student achievement as their foremost concern, while attitudes, if considered at all, are assessed by a very short questionnaire constructed by the investigator and administered as a post-test only. Examples of such research are the studies by Culp and Lagowski (19), Schmidt (80), Ibrahim (44), and Wassertheil (94) that were treated in the previous section.

A typical questionnaire, the one used by Schmidt (80) to evaluate his instruction, contained the following three items concerning attitude toward CAI (80, p. 12):

- Do you feel the computer information was an aid in problem solving?
- If you were given a choice, would you use this method to study calculus?
- Would you like to see this project extended to all the areas covered in calculus I?

Another example, the questionnaire used by Culp, asked the students to mark "strongly agree," "agree," "neutral," "disagree," or "strongly disagree" in response to each of the following questions (19, p. 361):

1. I enjoyed participating in the program.
2. The time required was well-spent.
3. I found that CAI was a definite aid to learning.
4. I would recommend continued development of CAI in organic chemistry.

Each of the investigators, Culp, Lagowski, Schmidt, Ibrahim, and Wassertheil, stated that students' responses to CAI were favorable. Nevertheless, it seems clear that a questionnaire of this kind can be expected to provide only a very rough idea of student attitudes

toward CAI.

A previously-described study conducted at the State University of New York (27) attempted to measure students' attitudes toward CAI, programmed instruction, and statistical laboratories by a seven-item questionnaire administered at the end of the course. Although CAI and programmed instruction seemed to evoke more favorable responses than the laboratories, the responses in all groups were extremely mixed. Because of the poor experimental controls, however, no solid conclusions can be drawn from this study.

There are several studies that have attempted more extensive attitudinal assessments than those described above. One such investigation is that by Brown and Gilman (14) in which high school physics students received instruction in physics by CAI under two different methods of feedback, or by printed programmed materials. For this study, the researchers developed a forty-item Likert style questionnaire that has since been used in experiments by other investigators. In this study, each of the CAI groups had a higher attitudinal score than the programmed text group, that took a form of the same questionnaire modified to measure attitude toward programmed instruction. The attitudes of the students toward CAI were not affected significantly by the method of feedback they had received.

The questionnaire developed by Brown was used with slight modifications in the study by Hall (35) that provided inservice training in mathematics to elementary school teachers through CAI.

The mean score for all students completing the questionnaire was 155.2, where a score of 126 is considered neutral and a higher score indicates a higher opinion of CAI. One point to be considered is the fact that the attitude survey was administered by the computer, and at one study center the students were required to follow a special set of instructions to sign on to the test. Of the 129 students who were given those instructions, 13 students failed to follow them and hence did not take the test. It seems likely to this investigator that students who are more "machine-shy" than others would be less likely to follow new instructions for taking the survey, and these "machine-shy" people might reasonably be expected to have attitudes toward CAI that are different from the attitudes of other students. Hence the students who completed the survey may be a biased sample of the group as a whole.

Roid (77), in a study in introductory psychology that was described earlier, used a semantic differential technique to measure attitude change. The test was administered on a pre-test, post-test basis, and the only changes reported were "on the understandability factor for the words 'computer' and 'computer programmer'" (77, p. 4). Both the CAI group and the group using the programmed text evaluated their media positively, and there was no significant difference between the attitudes of the two groups. There was also an indication that in the CAI group, the attitude change varied with the student's performance.

Longo (58), in his study of CAI in electronics training,

administered a twenty-two-item Likert questionnaire to the CAI students after their first week of CAI and again after a week of traditional instruction. On both administrations, the students displayed a favorable attitude toward CAI. They indicated a preference for CAI over traditional instruction on both administrations of the test, but their preference was by a smaller margin on the second administration.

In an experiment dealing with college students in remedial math, Judd et al. (48) separated students into five groups, and each group was given CAI with a different amount of control over the topics studied and over the number of items within a topic. Attitudes, measured by an eleven-item questionnaire, seemed to be slightly positive, but there were no consistent patterns for the different groups. Increased learner control did not lead to better student attitudes, nor did it affect achievement in mathematics. It should be mentioned that this experiment was troubled by a high experimental mortality and by program "bugs."

Investigators who were primarily interested in attitudinal changes as a result of CAI were Melnick et al. (65), Mathis et al. (62), and Sherman and Klare (83). In the study by Melnick et al. (65), students received three hours of lecture and two hours of CAI. Students were given a pre-test and a post-test measuring attitudes toward CAI, and the investigator states that "in 18 out of 20 comparisons there was a change toward a more favorable view of computers, and 14 of these 18 differences were statistically

significant" (65, p. 1). In comparing this experiment to others in this section, one should remember that this questionnaire was an attempt to measure attitudes toward the computer rather than attitudes toward CAI. Furthermore, the lecture part of the course in addition to the CAI experience probably contributed to this change in attitude, because in the first lecture period the instructor "tries to convince students that computers are a Good Thing, or at least relevant to their lives" (65, p. 2).

Using 64 students from a general psychology class at Florida State University, Mathis et al. (62) randomly assigned each student to one of two control groups or two experimental groups. One experimental group and one control group were given a pre-test on attitudes toward CAI, after which the two experimental groups received 45 minutes of CAI covering either new or familiar material in psychology, and the control groups spent 45 minutes reading either new or familiar material in psychology. All students were then given a post-test on attitude toward CAI. Pre-testing attitudes, Mathis found, was of little importance in this study, and he suggests that a control group for pre-testing attitudes is unnecessary. Exposure to CAI did change attitudes toward CAI in a positive direction, and students who studied familiar material by CAI displayed the greatest increase in positive attitude toward CAI.

Sherman and Klare (83) tested the attitude toward CAI of adult basic education students with no exposure to CAI, and the attitude toward CAI of a comparable group of students who had been given 15

minutes of CAI. The test was a fourteen-item questionnaire in which students were asked to agree or disagree with statements such as "it would be interesting to work by computer" (83, p. 6). Students who had worked on the computer had a significantly more positive attitude toward CAI than students who had no exposure to CAI.

#### Effects of CAI on Attitudes Toward Math

Attitudes toward mathematics, particularly those attitudes held by prospective primary teachers, have been an object of concern of educators throughout the years (23, 97), as researchers have tried to improve attitudes toward mathematics through such innovations as mathematics laboratories (40), enrichment topics (15, 55, 92), programmed instruction (7), and novel approaches to familiar topics (26). Nevertheless, very few studies employing CAI in mathematics have considered the possible effect of CAI on attitudes toward mathematics. Cole (16), Holoien (43), Ibrahim (44), and Hall (35), have completed such investigations, however.

Cole (16), who used CAI to improve basic math skills of high school students, was able to report a significant gain in attitude toward math of students using CAI. Although he used a control group of traditionally instructed students in his experiment, he does not report whether or not their attitude toward math also changed as a result of their coursework.

In his experiment in introductory calculus, Holoien (43)



randomly divided each of two classes into an experimental group that used computer programming in their study of calculus, and a control group that did not. In pre-test, post-test administrations of an attitude inventory, attitudes toward math were shown to be unaffected by the use of CAI.

Ibrahim (44), in a study described previously, used four groups of calculus students with a varying amount of CAI and traditional instruction. The group receiving four hours lecture by CAI and two hours problem-solving by CAI, and the two groups that did not use CAI all showed no significant changes in attitude toward mathematics. In the group receiving four hours CAI lecture and two hours of problem-solving by a teacher, however, the students' attitude toward math declined significantly.

The study by Hall (35) that gave inservice training to primary teachers also employed an attitude toward mathematics questionnaire as a pre-test and a post-test. In this experiment, the mean attitude scores on the post-test showed a slight but non-significant increase over the pre-test scores. It may well be that at least in an area such as mathematics, "measurable changes in attitude generally demand a longer time span than the seven weeks of this project" (35, p. 26).

In reviewing the attitudinal studies, one conclusion is that student reaction to CAI is generally positive, and that exposure to CAI is likely to improve student attitudes toward CAI. Although several studies have attempted to measure the effects of

CAI on attitudes toward mathematics, no clear patterns have been established in this field of research.

In the area of mathematics achievement, several studies have shown significant differences favoring traditional instruction or printed programmed instruction over CAI, while a few more studies have shown significant differences favoring CAI over traditional instruction. Nevertheless, most studies have failed to demonstrate a significant difference in achievement between students receiving CAI and students receiving an alternate method of instruction. It would seem, then, that CAI is not certain to be more effective or less effective than other instructional methods in a specific educational situation; instead, the relative success of CAI is likely to be determined by such factors as the quality of the individual computer programs, the adaptability of the course content to the chosen CAI format, the dependability of the system's hardware, and the personality characteristics of the individual learner.

## METHODS AND PROCEDURES

The main objectives of this study were to develop six CAI lessons and to evaluate the lessons' effectiveness in teaching mathematical content, in changing the student's attitude toward CAI, and in changing the student's attitude toward mathematics. Consequently, this chapter will first describe the development of the CAI lessons and the selection of appropriate instruments to evaluate these materials. The chapter concludes with a description of the experimental procedure and a description of the statistical treatment of the data.

## Development of the CAI Materials

Because the author wanted to expose students to CAI regularly throughout the school quarter with a minimum disruption of the standard classroom routine, the first step in writing the lessons was to select six topics that each required approximately one class period for lecture presentation, and that occurred at reasonable intervals from the second week of the quarter through the ninth week of the quarter. (Winter quarter at Iowa State is a school term consisting of ten weeks of instruction and one week of final examinations.) The range of topics was also narrowed somewhat by keyboard limitations, since the keyboard character set does not provide for an easy representation of such prospective topics as Venn diagrams or ancient numeration systems, for example. Additionally, an attempt was made to select topics that students in

previous quarters had found difficult, so that achievement gains that might occur are not likely to have resulted from simply reading the textbook. An outline of the content selected for each of the six programs may be found in appendix A.

All units in this study were written in CPS (Conversational Programming System), a time sharing system that the developers describe as "a means by which many people can use a large computer simultaneously while each has the illusion that the computer is responding to him alone" (45, p. 1). To use this system, a student dials the number assigned to CPS on a telephone that is used in conjunction with the computer terminal. The terminal used in this study is an IBM 2741 terminal, a device that looks like an ordinary typewriter. The student who uses one of the CAI units in this experiment first types a standard message on the terminal to identify himself, and then types instructions that cause the computer to load and to execute the appropriate lesson.

The lessons themselves are a series of statements and questions similar to the frames in a programmed instruction textbook. For each frame, the computer types a question or a series of statements followed by a question. After the student types his response to the question, the computer compares this response against a set of answers supplied by the programmer, and informs the student whether or not his response is correct. If the student is incorrect, the computer types the correct answer with an explanation, and in some frames it also types a message appropriate for the particular wrong

answer that the student has supplied. The student is then given a different question that may be of the same type or of a different type, depending on the student's past performance. A student who makes frequent errors, then, will be given more questions than a student who continually responds correctly.

A frame taken from the first lesson, for example, informs the student that in the base twelve system the letter "t" will be used to represent ten, and the student is asked to guess which symbol would be used for the number that directly follows "t." A student who responds with the correct answer, "e," will receive the reply, "Yes, e will be used to represent eleven in the base twelve system." If the student responds with a letter other than "e," the computer will type the message, "Very interesting. That symbol could be used, but we usually use the letter e instead." The most common wrong answer, however, is the numeral "11," so a student who makes that response will be told, "No, 11 can't be used, since  $11(\text{twelve}) = (1 \times 12) + (1 \times 1) = 13(\text{ten})$ . Instead, we will use the letter e."

In writing the actual computer programs, the major step was to write a series of instructions that would accept a student's response, eliminate any blank spaces in the response, capitalize any lower case letters in the response, and then match the response with a list of correct answers and type an appropriate message to the student, and branch to the next frame. Since this same set of instructions would be used in each frame, all that the author need do to write a new frame is to supply the following material: a

question; a set of answers that are to be counted as correct; a message to be typed in case the student's answer is not correct; and the number of the next frame the student is to work. In many frames, the author also supplies a list of anticipated wrong answers along with a message the computer is to type in response to each of those answers, and most frames also contain statements of information for the student and a counter to determine which question the student should receive next. A typical student would work approximately 17-20 frames in each of the CAI units.

To correct possible mistakes in the lessons and to identify parts of the lessons in which wording was unclear or confusing, the investigator arranged to have each of the 80 elementary education majors in her section of math 190 use one of the CAI lessons during fall quarter, 1971. On the basis of this trial use, several mistakes were eliminated in various programs and one program was rewritten to shorten the time required for computer response in a segment of that program. Also, observation of these students provided a basis for writing the set of student instructions for using the computer.

#### Evaluation Instruments

The questionnaire measuring attitude toward CAI is a modified version of a questionnaire developed by Brown at Pennsylvania State University (68). Brown constructed his forty-item questionnaire largely on the basis of written comments of students and observations

of students who had used CAI as a part of their coursework, and he reports the reliability of the instrument as .885 (68, p. 101). The author of the present paper, however, judged 15 questions on the Brown instrument to be inappropriate for this investigation, whereas four questions that the author wanted to ask were not included in the original questionnaire. The form of the questionnaire used in this study, which is printed in Appendix C, contains 25 items from Brown's questionnaire and 4 items, numbers 6, 7, 9, and 16, that were constructed by the investigator. Since many items on this questionnaire are appropriate only for students who have experienced CAI, this form was used as a post-test for the experimental group, and a second form of the questionnaire was constructed by making appropriate changes in the wording of this post-test form, usually changing only the verb tense. This new form, used as a pre-test for all students participating in the experiment and as a post-test for students in the control group, is printed in Appendix B.

Each questionnaire lists five responses, "strongly disagree," "disagree," "uncertain," "agree," and "strongly agree," for each item, but eleven of the items are worded positively and eighteen of the items express a negative attitude toward CAI. Each item is scored on a five-point basis with items expressing a positive attitude toward CAI scored as follows: 1 point for marking "strongly disagree"; 2 points for marking "disagree"; 3 points for marking "uncertain"; 4 points for marking "agree"; and 5 points for marking

"strongly agree." Scoring is reversed on items expressing a negative attitude toward CAI. In this way, a theoretically neutral attitude would be represented by a score of 87 (3x29), and the possible extreme scores are 29, expressing a negative attitude toward CAI, and 145, expressing a positive attitude toward CAI.

To measure attitude toward mathematics, a scale developed by Aiken and Dreger in (1) was selected. On this test, the authors claim a reliability coefficient of .94, and they also state that "a test of independence between the scores on the attitude scale and the scores on four items designed to measure attitudes toward academic subjects in general suggested that attitudes specific to mathematics were being measured" (3, p. 20).

This test consists of 20 items, 10 of which are positively worded and 10 negatively worded. Like the items on the Brown questionnaire, these are Likert items with five responses from "strongly disagree" to "strongly agree." Again, scoring is done on a five point basis so that the most negative attitude score is 20 (1x20), a neutral score is 60 (3x20), and the most positive attitude score is 100 (5x20).

Since an objective of this study was to measure very specific achievement, achievement in the material presented by the CAI units, the investigator found it necessary to construct her own achievement test. To provide a wider range of questions than were likely to come from one individual and to lessen the possibility of bias, however, sample tests and quizzes covering the topics



presented in the CAI units were obtained from Dr. Wilfred Barnes, Dr. Clarence Lindahl, Dr. James Peake, and Mrs. Phyllis Townswick, all of whom had previously taught math 190. Of the 24 items that comprise the achievement test, 16 are slightly modified items from previous tests of these instructors, and eight are items constructed by the investigator. There are four items worth one point each that deal with each of the six CAI units, so scores could vary from 0 through 24. Although six items are multiple-choice questions, the remaining items are short-answer or completion items.

After a final set of 24 items was selected, the investigator constructed a second set of 24 items by changing the wording or the specific numbers or key words in each of the first set of items. At this point, then, there were 24 pairs of comparable items, so one item in each pair was randomly assigned to the pre-test, form A of the achievement test, while the remaining item was assigned to the post-test, form B. The two forms of the test may be found in Appendix D and Appendix E respectively.

To fulfill a secondary objective of the experiment, a short questionnaire was written by Dr. Rex Thomas, Assistant Professor of Computer Science at Iowa State. The questionnaire, which contained four items expressing dissatisfaction with the CAI experience and two items expressing a positive evaluation of the experience, was constructed as a means of assessing attitude changes that may take place after each administration of CAI. After their first lesson, sixteen of the thirty-two students in

the experimental group were randomly selected to complete this questionnaire, which reads as follows:

1. The computer acted like a person talking to me.  

strongly	disagree	uncertain	agree	strongly
disagree				agree
2. I was afraid I would damage the machine.  

strongly	disagree	uncertain	agree	strongly
disagree				agree
3. I felt I understood the material presented in this lesson.  

strongly	disagree	uncertain	agree	strongly
disagree				agree
4. The computer was slow in responding to my answers.  

strongly	disagree	uncertain	agree	strongly
disagree				agree
5. The mechanics of using the computer are very complicated.  

strongly	disagree	uncertain	agree	strongly
disagree				agree
6. I was frustrated by machine problems on this lesson.  

strongly	disagree	uncertain	agree	strongly
disagree				agree

After each subsequent lesson, the same sixteen students were asked to complete a questionnaire containing the previous six items in addition to a seventh item, "this CAI lesson was more enjoyable than the previous one." The same five responses listed for the other items were also printed for this last question.

### Experimental Procedure

Two hundred forty-three students enrolled in math 190 in winter quarter, 1972, took the pre-tests in attitude toward CAI, attitude toward math, and achievement in math during the first class period of the quarter. By using a table of random numbers, a stratified random sample was drawn to form an experimental group and a control group. Each group consisted of 16 elementary education majors (15 females and one male), and 16 students not majoring in elementary education (10 females and six males). Although the groups were thus balanced by sex, no comparisons between sexes were made because of the relatively small number of male subjects. Students not selected for either the experimental group or the control group were required to take the post-tests also, but their scores were not used in the evaluation.

The basic reason for selecting a control group instead of using all remaining students as a control group is that such a procedure would involve the comparison of unlike groups. Since this study is primarily interested in prospective primary teachers, it was necessary to use all of the 32 elementary education majors enrolled in the course as subjects in the study. Yet because math 190 is a course that is more clearly relevant to the chosen career of these students than to the careers of students with other majors, one would expect the reaction of elementary education majors to the course to be different from the reaction of students with other

majors. If the experimental group and the control group had a different proportion of elementary education majors, then, it is possible that any significant differences would reflect the different composition of the groups rather than the experimental variable. Furthermore, limits on facilities and computer time necessitated a reasonably small experimental group, and the results of an analysis of covariance obtained by the regression technique may be biased when unequal sample sizes are used (52, p. 19). Hence it was desirable to use an experimental group and a control group of equal size or nearly equal size.

Students in the experimental group were told that on six specified class days during the quarter they were not to attend class. Instead, they were required to use the CAI unit covering the same topic as that day's class lecture. Since the programs could be used at any time, students were not limited to a particular day or class hour in which they could work a specific program. To make certain that students were actually using the programs, they were required to turn in the IBM sheet from the typewriter terminal after each lesson, but they were given assurance that the quality of their performance on the computer would have no effect on their course grade.

During the first CAI session, the investigator was present to distribute written instructions on using the computer, and to help students who were having difficulty following the instructions. After the first session, however, students used the computer without

the author's help. At the first session and throughout the course, the investigator made every attempt to avoid association or personal involvement with the computer programs. Students were told simply that these programs had been written for use in math 190, but they were not told that the programs had been written by their instructor.

#### Treatment of Data

In addition to developing the CAI units, the objectives of this study were to determine if the use of CAI as a part of an undergraduate mathematics course could change the student's attitude toward CAI or his attitude toward mathematics, and to compare gains in mathematics achievement made by CAI students with gains made by traditionally-instructed students. To satisfy these objectives, the following null hypotheses were stated:

1. There is no significant change in the control group's attitude toward CAI as a result of taking math 190.

2. There is no significant change in the CAI group's attitude toward CAI as a result of taking math 190.

3. There is no significant difference in the control group's attitude toward CAI and the CAI group's attitude toward CAI as measured by the post-test when original pre-test differences have been controlled.

4. There is no significant change in the control group's attitude toward mathematics as a result of taking math 190.

5. There is no significant change in the CAI group's attitude

toward mathematics as a result of taking math 190.

6. There is no significant difference in the control group's attitude toward mathematics and the CAI group's attitude toward mathematics as measured by the post-test when initial pre-test differences have been controlled.

7. There is no significant change in the control group's achievement in mathematics as a result of taking math 190.

8. There is no significant change in the CAI group's achievement in mathematics as a result of taking math 190.

9. There is no significant difference in the control group's achievement in mathematics and the CAI group's achievement in mathematics as measured by the post-test when original pre-test differences have been controlled.

Hypotheses 1, 2, 4, 5, 7, and 8 each require a t-test using the statistic  $t = \frac{\bar{X}_2 - \bar{X}_1}{\sqrt{\frac{S_1^2 + S_2^2 - 2r_{12}S_1S_2}{N}}}$

$$\sqrt{\frac{S_1^2 + S_2^2 - 2r_{12}S_1S_2}{N}}$$

For hypothesis 1,  $\bar{X}_1$  = mean score on pre-test of attitude toward CAI of the control group

$\bar{X}_2$  = mean score on post-test of attitude toward CAI of the control group

$S_1$  = standard deviation of the control group's pre-test scores on attitude toward CAI

$S_2$  = standard deviation of the control group's post-test scores on attitude toward CAI

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on attitude toward CAI for the control group

$N$  = number of students in the control group

Using hypothesis 2,  $\bar{X}_1$  = mean score on pre-test attitude toward CAI of the CAI group

$\bar{X}_2$  = mean score on post-test attitude toward CAI of the CAI group

$S_1$  = standard deviation of the CAI group's pre-test scores on attitude toward CAI

$S_2$  = standard deviation of the CAI group's post-test scores on attitude toward CAI

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on attitude toward CAI for the CAI group

$N$  = number of students in the CAI group

With hypothesis 4,  $\bar{X}_1$  = mean score on pre-test of attitude toward mathematics of the control group

$\bar{X}_2$  = mean score on post-test of attitude toward mathematics of the control group

$S_1$  = standard deviation of the control group's pre-test scores on attitude toward mathematics

$S_2$  = standard deviation of the control group's post-test scores on attitude toward mathematics

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on attitude toward mathematics for the control group

$N$  = number of students in the control group

In hypothesis 5,  $\bar{X}_1$  = mean score on pre-test of attitude toward mathematics of the CAI group

$\bar{X}_2$  = mean score on post-test of attitude toward mathematics of the CAI group

$S_1$  = standard deviation of the CAI group's pre-test scores on attitude toward mathematics

$S_2$  = standard deviation of the CAI group's post-test scores on attitude toward mathematics

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on attitude toward mathematics for the CAI group

$N$  = number of students in the CAI group

Using hypothesis 7,  $\bar{X}_1$  = mean score on pre-test of math achievement of the control group

$\bar{X}_2$  = mean score on post-test of math achievement of the control group

$S_1$  = standard deviation of the control group's pre-test scores on achievement in math

$S_2$  = standard deviation of the control group's post-test scores on achievement in math

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on achievement in math for the control group

$N$  = number of students in the control group

For hypothesis 8,  $\bar{X}_1$  = mean score on pre-test of math achievement of the CAI group

$\bar{X}_2$  = mean score on post-test of math achievement of the CAI group

$S_1$  = standard deviation of the CAI group's pre-test scores on achievement in math

$S_2$  = standard deviation of the CAI group's post-test scores on achievement in math

$r_{12}$  = correlation coefficient between the pre-test and post-test scores on achievement in math for the CAI group

$N$  = number of students in the CAI group



Hypotheses 3, 6, and 9 require an analysis of covariance procedure. Each case takes into account the student's major (elementary education or not elementary education) and treatment (CAI or control group), and each hypothesis uses the following model:

$$Y_{ijk} = U + T_i + C_j + TC_{ij} + B(X_{ijk} - \bar{X}) + E_{ijk} \quad \begin{array}{l} i = 1, 2 \\ j = 1, 2 \\ k = 1, 2, \\ \dots, 64 \end{array}$$

For each of these three hypotheses,  $T_i$  = effect of the  $i$ th treatment

$C_j$  = effect of the  $j$ th major

$TC$  = effect of interaction  
between treatment and  
major

$B$  = regression coefficient of  
Y on X

and  $E_{ijk}$  = residual associated with  
the  $k$ th individual in  
treatment  $i$  and major  $j$

In hypothesis 3, the remaining variables in the model are identified as follows:

$Y_{ijk}$  = post-test attitude toward CAI score of the  $k$ th  
individual in treatment  $i$  with major  $j$

$U$  = grand mean post-test score on attitude toward CAI

$\bar{X}$  = grand mean pre-test score on attitude toward CAI

$X_{ijk}$  = pre-test score on attitude toward CAI for the  $k$ th  
individual in treatment  $i$  with major  $j$

For hypothesis 6,  $Y_{ijk}$  = post-test attitude toward math score of  
the  $k$ th individual with treatment  $i$ ,  
major  $j$

$U$  = grand mean post-test score on attitude toward math

$\bar{X}$  = grand mean pre-test score on attitude toward math

$X_{ijk}$  = pre-test score on attitude toward math for the kth individual in treatment i and major j

Using hypothesis 9, the remaining variables are as follows:

$Y_{ijk}$  = post-test score on math achievement for the kth individual in treatment i with major j

$U$  = grand mean post-test score in math achievement

$\bar{X}$  = grand mean pre-test score in math achievement

$X_{ijk}$  = pre-test score on mathematics achievement for the kth individual in treatment i with major j

The secondary objectives of the study were to investigate the possible relationships among the three major variables in the study, and to measure any change in attitude that might occur after each CAI lesson. For each individual, there is a pre-test score on attitude toward CAI, on attitude toward math, and on achievement in math, and a post-test score on the same components. To complete the first objective, then, one would calculate the correlation coefficient between all possible pairs of these six variables.

To investigate the second problem, responses to a seven-item questionnaire administered after each CAI lesson must be analyzed. As a means of insuring that this questionnaire does not interfere with the longer questionnaire on attitude toward CAI, an analysis of covariance is done to test the following null hypothesis: there is no significant difference in the post-test scores

measuring attitude toward CAI of those CAI students who completed the short attitudinal forms and those CAI students who were not given the short attitudinal forms. If the null hypothesis is rejected, the data supplied by students who completed the short attitudinal questionnaires must be eliminated from the analysis of the main questionnaire on attitude toward CAI. Whether or not the null hypothesis is rejected, the mean response to each of the questions will be calculated for each CAI lesson, and the means will be graphed against the lesson numbers to determine what trends may be indicated.

## FINDINGS

This chapter contains the results of statistical tests that were performed on the data collected throughout the experiment. The results have been organized as follows: results of tests of the main hypotheses; analysis of the relationship between variables in the study; and analysis of student response to the short attitudinal questionnaires.

## Tests of the Main Hypotheses

As previously stated, this study is primarily concerned with changes in attitude toward CAI, changes in attitude toward mathematics, and changes in mathematics achievement that may result from the use of the CAI units as a part of a standard mathematics course. The first two major hypotheses, then, deal with attitude toward CAI, and may be stated in the null form as follows:

1. There is no significant change in the control group's attitude toward CAI as a result of taking math 190.
2. There is no significant change in the CAI group's attitude toward CAI as a result of taking math 190.

For the first hypothesis, the calculated  $t$  value is 1.73, which is not significant at the .05 level. In testing the second null hypothesis, however, the calculated  $t$  value is 7.19, which is significant beyond the .01 level. Thus the first null hypothesis cannot be rejected at the .05 level, but the second null hypothesis is rejected at both the .05 level and the .01 level.

The third hypothesis, also stated in the null form, is written as follows:

3. There is no significant difference in the control group's attitude toward CAI and the CAI group's attitude toward CAI when initial pre-test differences have been controlled.

To investigate this hypothesis, an analysis of covariance, shown in Table 1, was done using two classifications for treatment (CAI or traditional instruction), and two classifications for curriculum (elementary education or not elementary education), and using the post-test attitude toward CAI scores as the covariate. The analysis reveals that differences between treatments are significant beyond the .01 level, so null hypothesis 3 is rejected at the .05 level and at the .01 level. The F values associated with differences between curriculums and with interaction between treatment and curriculum, however, are not significant at the .05 level.

Although both the experimental group and the control group had 32 students at the start of the experiment, data on four students in the experimental group and two students in the control group are not available for analysis. Of the four students missing from the experimental group, two students failed to complete the experiment because they dropped math 190 from their class schedules, one student failed to take the post-test, and one student failed to complete the last five CAI lessons. The two students missing from the control group followed a similar pattern; one student dropped

Table 1. Analysis of covariance of attitudes toward CAI using pre-test attitude toward CAI scores as a covariate

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	3336.52	3336.52	27.89**
Curriculum	1	1.16	1.16	.01
Treatment x curriculum	1	26.26	26.26	.22
Residual	53	6340.56	119.63	
Total	56	9704.50		

\*\*Significant at the .01 level

the course, and one student failed to take the post-test.

The second series of hypotheses, those dealing with attitudes toward mathematics, may be stated in the null form as follows:

4. There is no significant change in the control group's attitude toward mathematics as a result of taking math 190.

5. There is no significant change in the CAI group's attitude toward mathematics as a result of taking math 190.

6. There is no significant difference in the control group's attitude toward mathematics and the CAI group's attitude toward mathematics when initial pre-test differences have been controlled.

The calculated t statistic for the fourth hypothesis is 2.10, which is significant at the .05 level. For the fifth hypothesis,

the calculated t value of 1.72 approaches significance, but it fails to be significant at the .05 level. Hence the fourth null hypothesis is rejected at the .05 level; the fifth null hypothesis cannot be rejected at the .05 level.

In hypothesis 6, treatments and curricula are as described in hypothesis 3, but the criterion variable is the post-test score on the attitude toward mathematics questionnaire, and the covariate is the pre-test score on the attitude toward mathematics questionnaire. In an analysis of covariance, presented in Table 2, no significant differences are found between treatments. The effects of curriculum and of interaction between treatment and curriculum are also non-significant. Null hypothesis 6 can therefore not be rejected at the .05 level.

Table 2. Analysis of covariance of attitude toward mathematics using pre-test attitude toward mathematics scores as a covariate

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	4.85	4.85	.09
Curriculum	1	.86	.86	.02
Treatment x curriculum	1	144.20	144.20	2.55
Residual	53	3001.40	56.63	
Total	56	3151.31		

Hypotheses 7 and 8 were previously stated in the null form as follows:

7. There is no significant change in the control group's achievement in mathematics as a result of taking math 190.

8. There is no significant change in the CAI group's achievement in mathematics as a result of taking math 190.

With hypothesis 7, the calculated t statistic is 7.38, which is significant beyond the .01 level. Similarly, the calculated t-statistic for hypothesis 8 is 13.57, which is significant beyond the .01 level. Both null hypothesis 7 and null hypothesis 8 are thus rejected at the .05 level and at the .01 level.

The last major hypothesis under investigation was previously stated as follows:

9. There is no significant difference in the control group's achievement in mathematics and the CAI group's achievement in mathematics when initial pre-test differences have been controlled.

Again, an analysis of covariance was done using the post-test math achievement scores as the criterion variable and the pre-test math achievement scores as the covariate. The analysis, printed in Table 3, shows no significant differences between treatments, no significant differences between curricula, and no significant interaction between treatment and curriculum. Consequently, null hypothesis 9 cannot be rejected at the .05 level.

The means and standard deviations of each of the variables used in these hypotheses may be found in Table 4.



Table 3. Analysis of covariance of mathematics achievement using pre-test mathematics achievement scores as a covariate

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	22.34	22.34	1.24
Curriculum	1	10.89	10.89	.61
Treatment x curriculum	1	53.98	53.98	3.00
Residual	53	952.95	17.98	
Total	56	1040.16		

#### Relationships among Variables

The secondary objectives of the study were to investigate the possible relationships among the major variables and to determine the amount of change in student attitude that occurs after each CAI lesson.

In response to the question of relationships among the variables, the correlation coefficient for each possible pair of the six variables used in this study was calculated for students in the CAI group, and the correlation matrix is presented in Table 5. These correlation coefficients for students in the control group were also calculated, and are presented in Table 6.

Table 4. Means and standard deviations of the major variables in the study

Variable	CAI group mean	CAI group standard deviation	Control group mean	Control group standard deviation
Pre-test CAI attitude	90.04	11.86	87.53	13.18
Post-test CAI attitude	107.68	10.92	91.03	14.23
Pre-test math attitude	59.32	19.76	50.03	20.90
Post-test math attitude	61.75	17.82	53.47	19.01
Pre-test math achievement	8.36	3.58	7.77	2.85
Post-test math achievement	16.64	3.76	15.07	5.30

Table 5. Correlation coefficients between listed variables for students in the CAI group

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test atti- tude toward CAI	1	.353	.050	.039	.081	-.289
Post-test atti- tude toward CAI		1	.148	.044	-.095	-.289
Pre-test atti- tude toward math			1	.926**	.574**	.515**
Post-test atti- tude toward math				1	.531**	.517**
Pre-test achieve- ment in math					1	.614**
Post-test achieve- ment in math						1

\* Significant at the .05 level

\*\* Significant at the .01 level

Table 6. Correlation coefficients between listed variables for students in the control group

	Pre-test attitude toward CAI	Post-test attitude toward CAI	Pre-test attitude toward math	Post-test attitude toward math	Pre-test achievement in math	Post-test achievement in math
Pre-test atti- tude toward CAI	1	.675**	-.149	-.077	.167	-.028
Post-test atti- tude toward CAI		1	-.095	.029	.041	-.249
Pre-test atti- tude toward math			1	.904**	.512**	.314
Post-test atti- tude toward math				1	.600**	.295
Pre-test achieve- ment in math					1	.229
Post-test achieve- ment in math						1

\* Significant at the .05 level

\*\* Significant at the .01 level

## Response to the Short Attitudinal Forms

Before considering the results of the short questionnaires on attitude toward CAI, it was necessary to determine whether or not the use of these questionnaires had a significant effect on a student's post-test score on the major CAI questionnaire used in hypotheses 1-3. If such an effect were found, it would be necessary to perform the analysis of post-test attitude toward CAI scores without the scores of students who had completed the short questionnaires. The hypothesis under investigation was stated in the null form as follows: there is no significant difference in the post-test scores measuring attitude toward CAI of those CAI students who completed the short attitudinal questionnaires and those CAI students who were not given the short attitudinal questionnaires. Of the 16 students who were asked to complete the questionnaires, two students dropped out of the course and two students failed to return a complete set of questionnaires, so the analysis was done on the 12 students who had completed all of the short attitudinal questionnaires and the 13 students who had completed no short attitudinal questionnaires. Table 7 presents the results of an analysis of covariance with post-test CAI scores as the criterion variable, and with two treatments (short attitudinal questionnaires or no short attitudinal questionnaires) and two curricula (elementary education and not elementary education), using pre-test attitude toward CAI scores as the covariate. Since the F value associated with treatment differences is not

Table 7. Analysis of covariance of post-test attitude toward CAI scores of CAI students using pre-test attitude scores as a covariate

Source of variation	d.f.	S.S.	M.S.	F
Treatment	1	31.65	31.65	.31
Curriculum	1	8.33	8.33	.08
Treatment x curriculum	1	391.35	391.35	3.89
Residual	20	2014.25	100.71	
Total	23	2445.58		

significant, the null hypothesis cannot be rejected at the .05 level, so the scores of students who completed the short attitudinal questionnaire were included in testing hypotheses 2 and 3 of this study.

The mean response to each question in the short attitudinal questionnaires was calculated for each lesson, and the questions and means are printed in Figures 1-7. Figures 1-7 also contain graphs of the means against the lesson numbers.

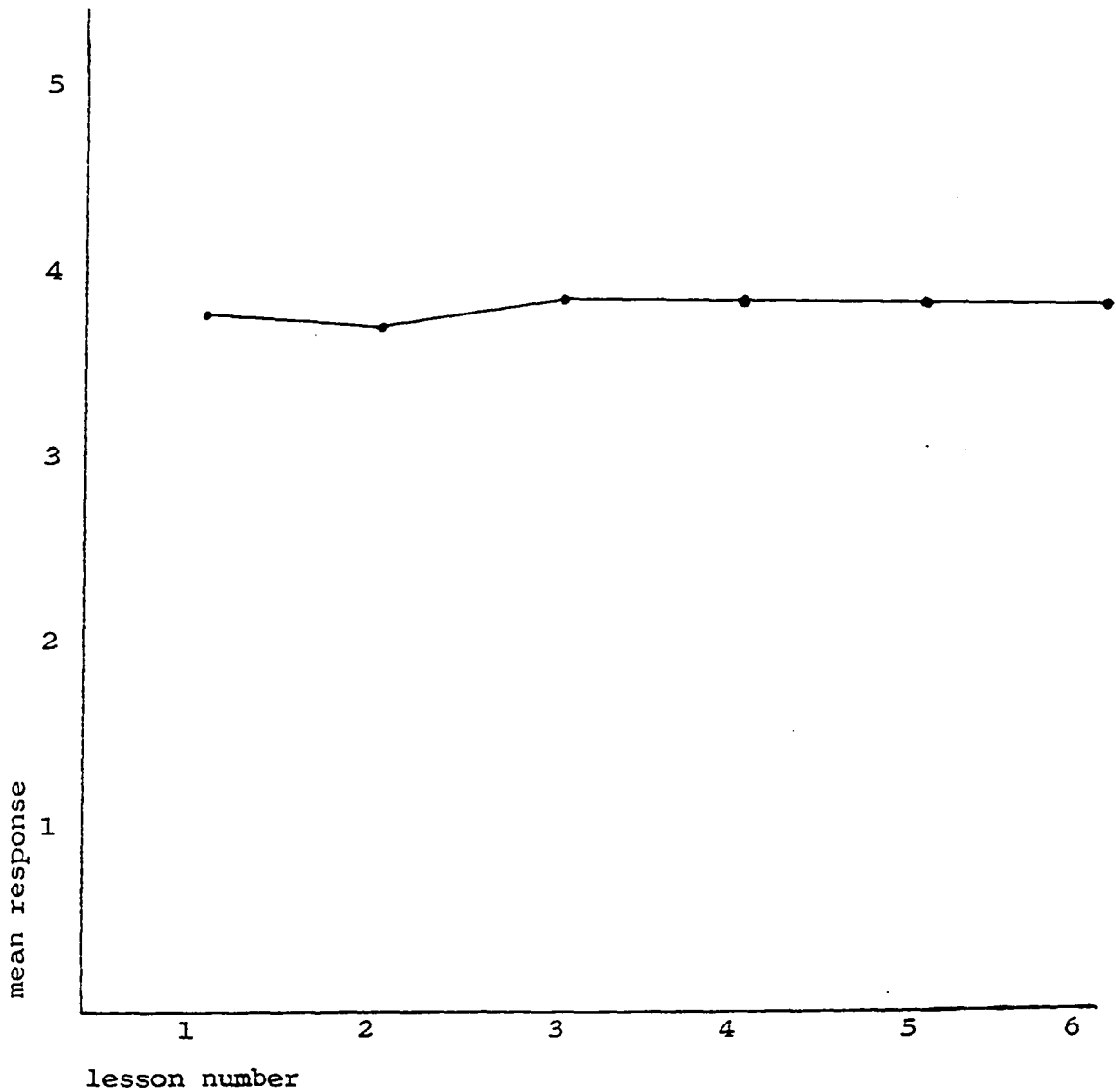


Figure 1. Mean response after each lesson to the following question:

1. The computer acted like a person talking to me.

1=strongly disagree    2=disagree    3=uncertain    4=agree    5=strongly agree

lesson 1	lesson 2	lesson 3	lesson 4	lesson 5	lesson 6
mean=3.75	mean=3.67	mean=3.83	mean=3.83	mean=3.83	mean=3.83

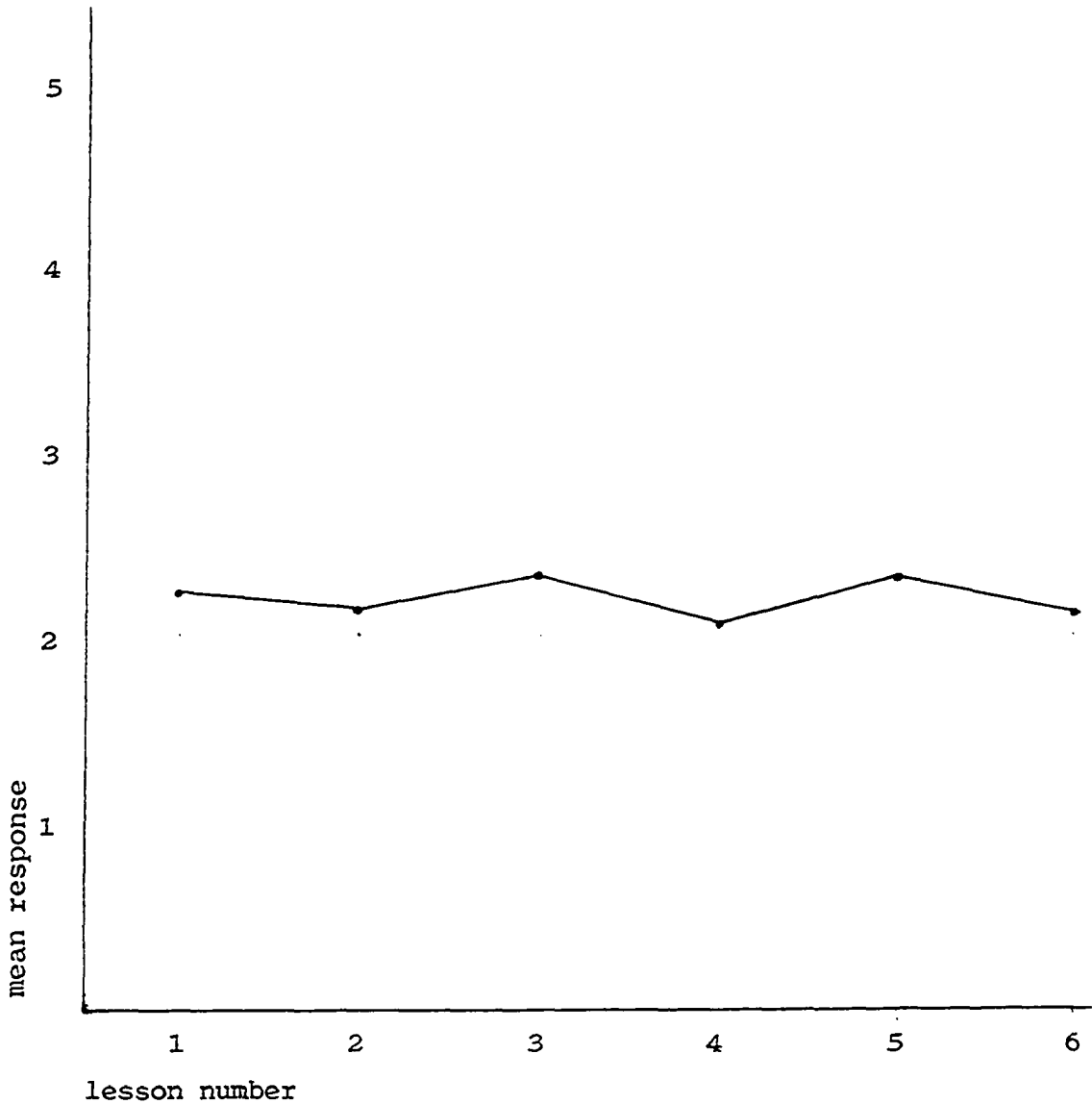


Figure 2. Mean response after each lesson to the following question:

2. I was afraid I would damage the machine.

1=strongly  
disagree

2=disagree

3=uncertain

4=agree

5=strongly  
agree

lesson 1  
mean=2.25

lesson 2  
mean=2.17

lesson 3  
mean=2.33

lesson 4  
mean=2.08

lesson 5  
mean=2.33

lesson 6  
mean=2.17



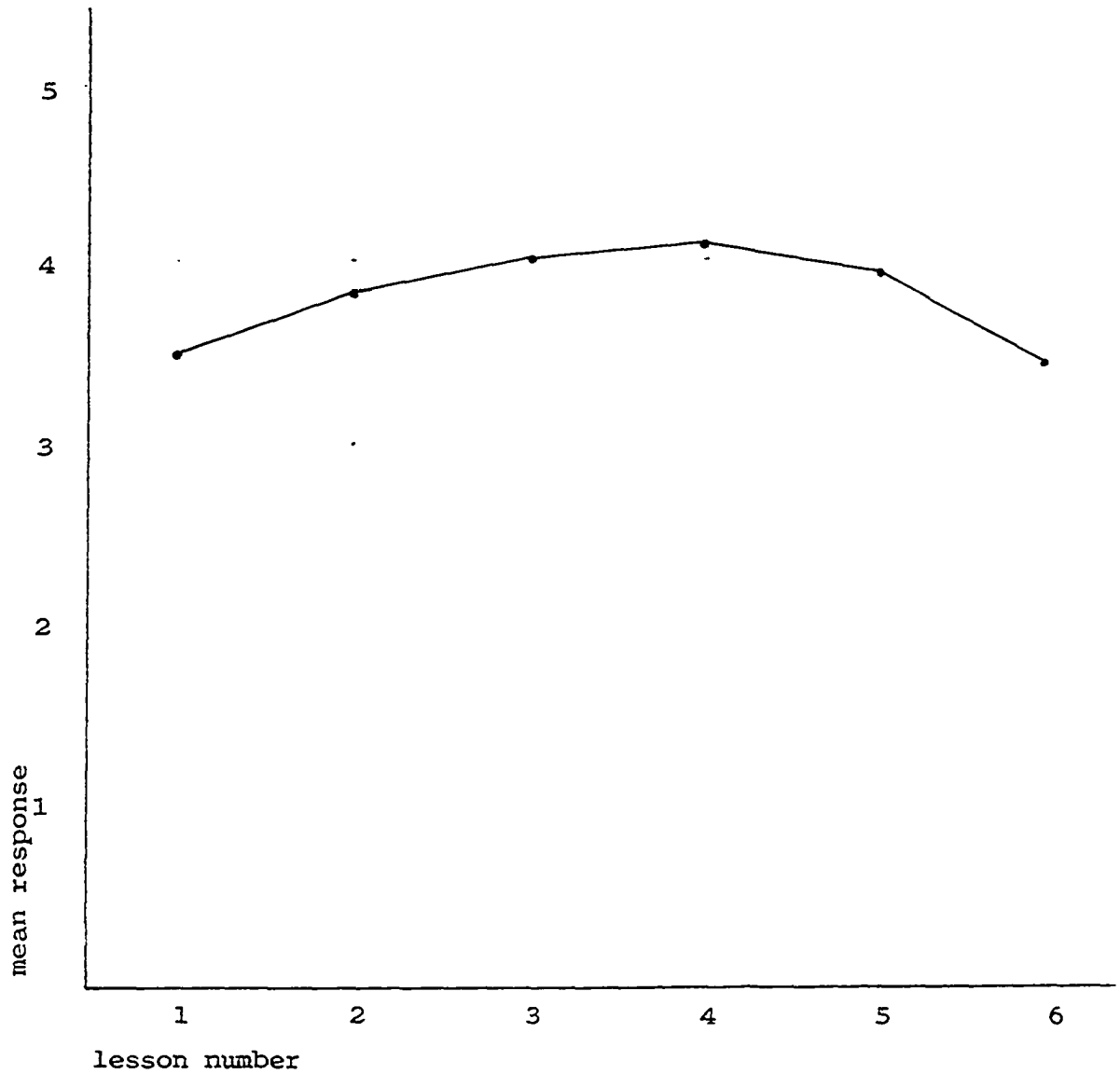


Figure 3. Mean response after each lesson to the following question:

3. I felt I understood the material presented in this lesson.

1=strongly  
disagree

2=disagree

3=uncertain

4=agree

5=strongly  
agree

lesson 1  
mean=3.50

lesson 2  
mean=3.83

lesson 3  
mean=4.00

lesson 4  
mean=4.08

lesson 5  
mean=3.92

lesson 6  
mean=3.41

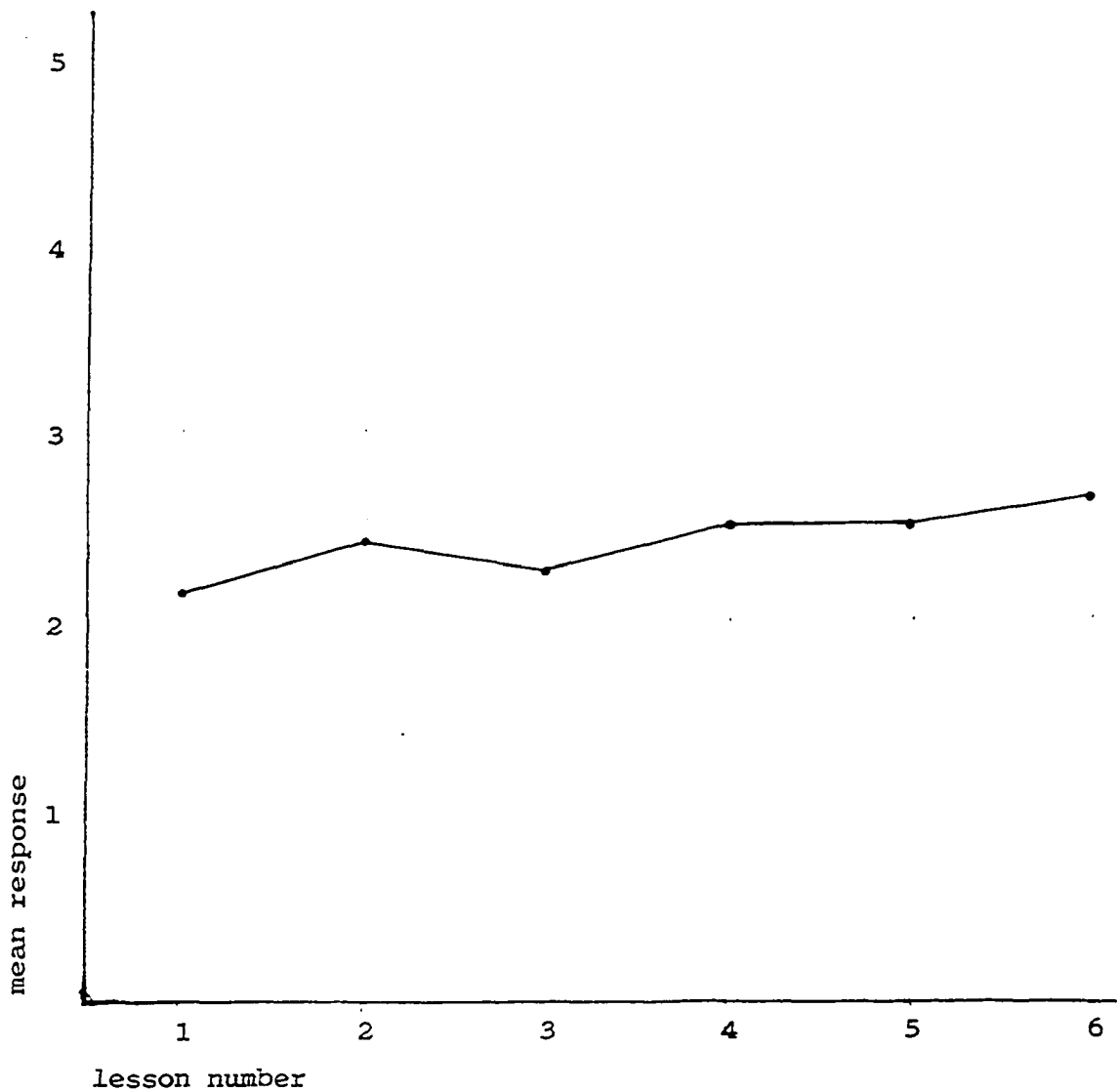


Figure 4. Mean response after each lesson to the following question:

4. The computer was slow in responding to my answers.

1=strongly disagree    2=disagree    3=uncertain    4=agree    5=strongly agree

lesson 1	lesson 2	lesson 3	lesson 4	lesson 5	lesson 6
mean=2.17	mean=2.42	mean=2.25	mean=2.50	mean=2.50	mean=2.67

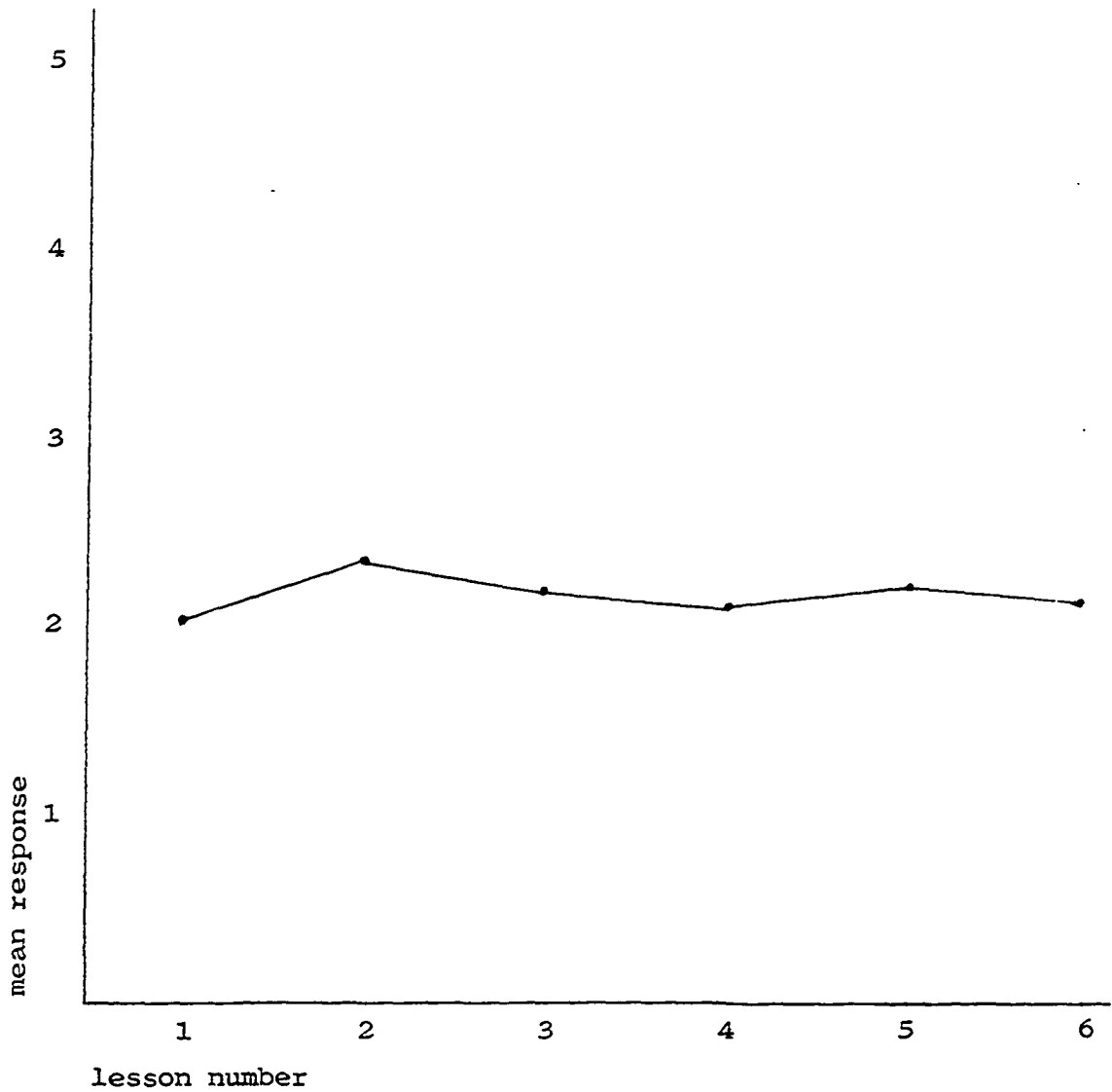


Figure 5. Mean response after each lesson to the following question:

5. The mechanics of using the computer are very complicated

1=strongly disagree    2=disagree    3=uncertain    4=agree    5=strongly agree

lesson 1	lesson 2	lesson 3	lesson 4	lesson 5	lesson 6
mean=2.00	mean=2.33	mean=2.17	mean=2.08	mean=2.17	mean=2.08

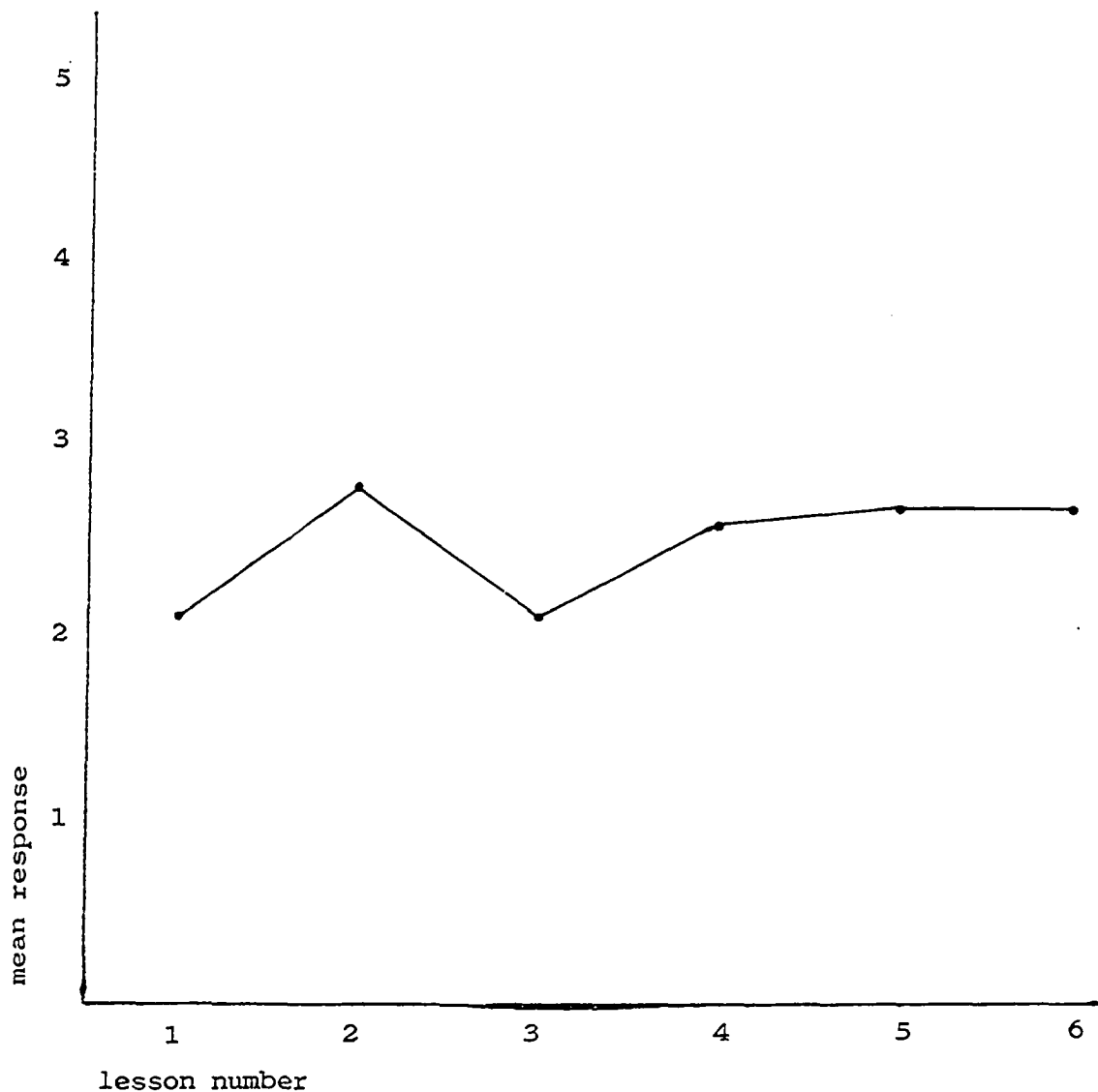


Figure 6. Mean response after each lesson to the following question:

6. I was frustrated by machine problems on this lesson.

1=strongly disagree    2=disagree    3=uncertain    4=agree    5=strongly agree

lesson 1	lesson 2	lesson 3	lesson 4	lesson 5	lesson 6
mean=2.08	mean=2.75	mean=2.08	mean=2.58	mean=2.67	mean=2.67

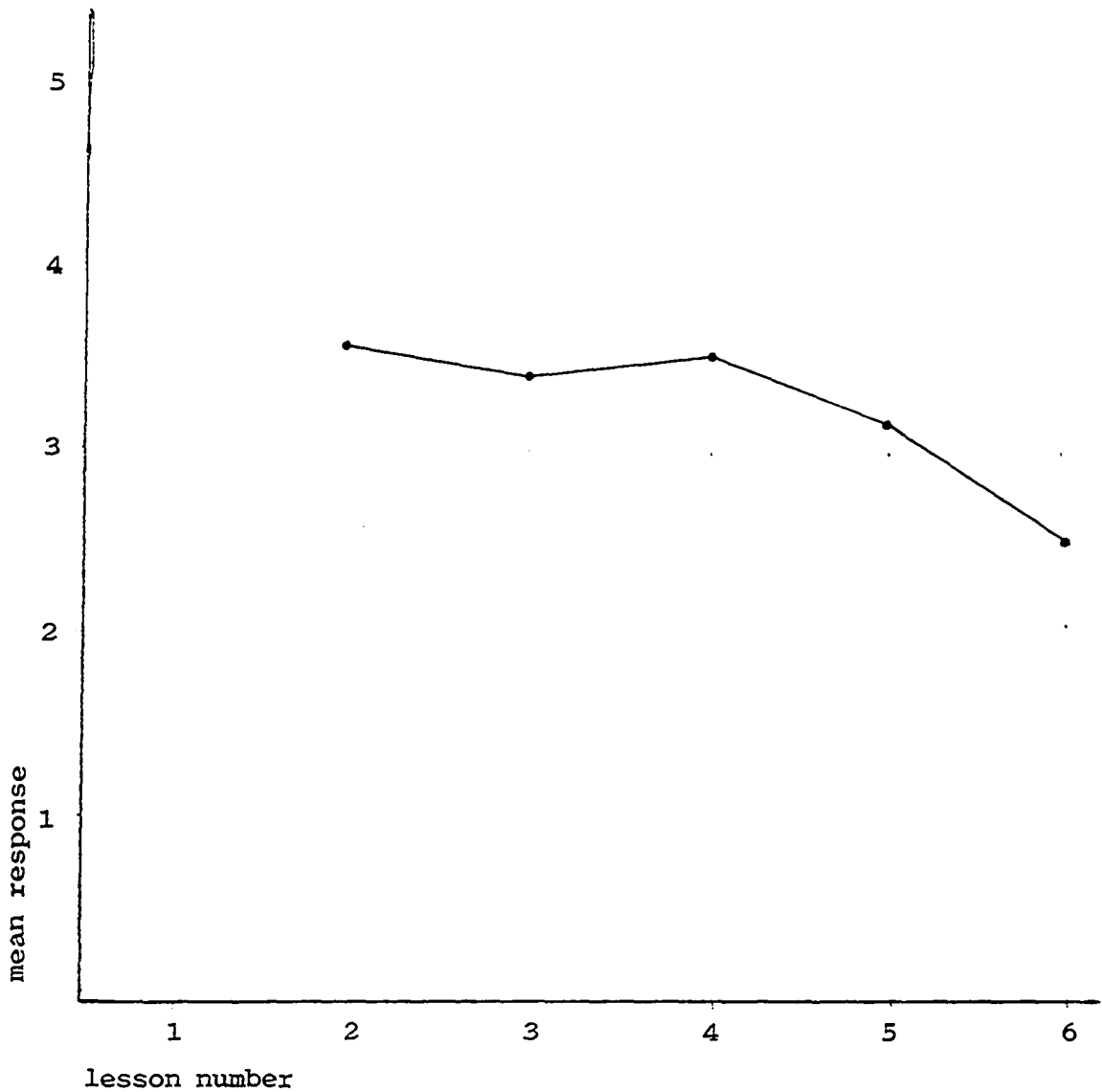


Figure 7. Mean response after each lesson to the following question:

7. This CAI lesson was more enjoyable than the previous one.

1=strongly disagree    2=disagree    3=uncertain    4=agree    5=strongly agree

lesson 2	lesson 3	lesson 4	lesson 5	lesson 6
mean=3.58	mean=3.42	mean=3.50	mean=3.17	mean=2.50

## DISCUSSION

The contention that a student's attitude toward CAI can be changed through the use of CAI as a part of his coursework was strongly supported by this study. First, the hypothesis of no significant change in the CAI group's attitude toward CAI was rejected at the .01 level, but the hypothesis of no significant change in the control group's attitude toward CAI could not be rejected at the .05 level. Also rejected at the .01 level was the hypothesis of no significant difference between the post-test attitude toward CAI of the CAI group and the control group when original pre-test differences are controlled. In other words, the CAI group and the control group did have different attitudes toward CAI at the end of the experiment; the CAI group's attitude toward CAI had changed significantly in the positive direction, while the control group's attitude had not changed significantly. This outcome is consistent with previous research in the subject (65, 62, 83).

In the area of attitude toward mathematics, however, conclusions are less clear. Attitudes toward mathematics improved in both the CAI group and the control group, but this change in the control group's attitude fell slightly above significance at the .05 level, and the change in the CAI group's attitude fell below significance at the .05 level. Nevertheless, an analysis of covariance controlling on pre-test attitudes toward mathematics scores revealed that the attitudes toward mathematics of the two

groups were not significantly different. It would seem, then, that there is not enough evidence to say that CAI is less effective than traditional instruction in changing attitudes toward mathematics, but the hope that CAI would lead to a greater attitude gain was definitely not substantiated.

This experiment also found no significant difference in mathematics achievement between students who had used CAI and students who received traditional instruction. Both groups made significant gains in math achievement between pre-test and post-test, and although gains made by the CAI group were greater than those of the control group, the difference between post-test scores of the two groups was non-significant when original pre-test differences were controlled. In assessing achievement, however, it should be noted that a typical student would complete each CAI unit in less time than the standard 50 minute class period. To the nearest minute, the mean time spent on each of the six CAI lessons is as follows: 42 minutes, 25 minutes, 17 minutes, 18 minutes, 26 minutes, and 25 minutes. For students receiving traditional instruction, the material contained in lesson three and lesson four required approximately 30 minutes of classroom lecture, but the material covered in each of the remaining lessons required 40 minutes of classroom lecture. Additionally, ten minutes of class time is allowed for questions, so the traditionally-instructed students spent 40 minutes on the topic of lesson three, 40 minutes on the topic of lesson four, and 50 minutes on the topic of each of the other four lessons.

Thus CAI may be credited with producing achievement gains comparable to those resulting from traditional instruction in less time than was required by traditional instruction.

In reviewing the correlation coefficients between the major variables in this study, a reasonable conclusion is that attitude toward CAI is unrelated to either attitude toward mathematics or achievement in mathematics. For both the CAI group and the control group, the correlation coefficient between pre-test attitude toward CAI and each of the four variables concerning mathematics, (pre-test attitude toward mathematics, post-test attitude toward mathematics, pre-test achievement in mathematics, and post-test achievement in mathematics), fails to reach significance at the .05 level. Similarly, there is no significant correlation between post-test attitude toward CAI and any of the four variables concerning mathematics in either the CAI group or the control group. Brown and Gilman (14) summarize research in this area by stating that "correlations between student attitude and performance measures tend to be positive but generally small, accounting for less than twenty per cent of the variance" (14, p. 41). In the present study, the correlation coefficient between post-test attitude toward CAI and post-test achievement in mathematics was actually negative for the CAI group, but quite small, (-.289), accounting for less than 9% of the variance. For a sample of 28 people, a correlation coefficient must be greater than .374 or less than -.374 to be considered significant. Similarly, with a group of 30 subjects a correlation



coefficient must be greater than .361 or less than  $-.361$  to be considered significant.

The fact that correlations between pre-test attitude toward math and post-test attitude toward math are extremely high in both the CAI group ( $r=.926$ ) and the control group ( $r=.904$ ) indicates that attitudes toward mathematics are firmly entrenched in both groups of students. On the other hand, the correlation between pre-test attitude toward CAI and post-test attitude toward CAI was much lower in both groups, although the correlation was highly significant in the control group ( $r=.675$ ) but non-significant in the CAI group ( $r=.353$ ). It seems reasonable that attitudes toward CAI are less firmly established than attitudes toward mathematics, and that consequently, attitudes toward CAI are more likely to change in the course of a school quarter.

The correlation coefficient between pre-test attitudes toward mathematics and pre-test achievement in mathematics are very similar in the CAI group ( $r=.574$ ) and in the control group ( $r=.512$ ). Although both coefficients are highly significant, they are still relatively small, since each coefficient accounts for less than one-third of the total variance.

For several variables, the correlation coefficients are quite different for the CAI group and the control group. In particular, the following three correlation coefficients are higher in the CAI group than in the control group: pre-test achievement in mathematics and post-test achievement in mathematics ( $r=.614$  for the CAI group,

$r=.229$  for the control group); post-test attitude toward mathematics and post-test achievement in mathematics ( $r=.517$  for the CAI group,  $r=.295$  for the control group); and pre-test attitude toward mathematics and post-test achievement in mathematics ( $r=.515$  for the CAI group,  $r=.314$  for the control group). That is, pre-test attitude toward mathematics and pre-test achievement in mathematics were larger factors in the CAI group than in the control group in determining post-test attitude toward mathematics and post-test achievement in mathematics. The fact that the correlation between pre-test and post-test math attitudes and the correlation between post-test math attitude and post-test math achievement are lower in the control group may be a reflection of the difference in math attitude gains between the control group and the CAI group; math attitudes in the control group changed significantly, but the change in math attitudes in the CAI group did not reach significance at the .05 level. Then too, these differences in correlation coefficients could simply emphasize the point that students react to CAI differently than they react to traditional instruction, but the specific reasons behind these differences have not yet been determined. It is also possible, of course, that these differences in correlation coefficients are just chance sampling variations.

From the short attitudinal questionnaires that were administered after each of the six CAI lessons, one may draw the following conclusions:

1. Students generally agreed that the computer acted like a

person talking to them, since the mean response to question one fell in the interval 3.67-3.83 for each of the lessons. (On this scale, 3 represents "uncertain," and 4 represents "agree.") Response to this question was remarkably stable; no student changed his response to this question after lesson number three.

2. Most students were not afraid that they would damage the machine, although the mean response to this question, question two, varied from lesson to lesson without a consistent pattern. The fact that students were more likely to mark "uncertain" in response to question two on the third lesson may have been a result of a change in procedure for using the machine, since this change in procedure went into effect just before most students attempted their third lesson.

3. Most students felt that they understood the material presented in each lesson. More students, however, felt uncertain or felt that they did not understand the material on the first lesson and on the last lesson than on any of the four remaining lessons. One explanation for the uncertainty in the first lesson is that students who were apprehensive about their first experience with CAI were therefore unable to concentrate on the material as well as they did on subsequent lessons. On the last lesson, on the other hand, the students' uncertainty could easily have resulted from the usual pressures that students encounter during the last week of scheduled classes in the school term, or it could be that students tend to become tired or bored with CAI after six lessons.

A final possibility is that the course material or the lessons themselves actually were more difficult to understand than the material in lessons 2 through 5. Repeating the experiment with the lessons in a different order would indicate the credibility of this last hypothesis.

4. Students usually disagreed with the statement that the computer was slow in responding to their answers, but the means tended more toward the "uncertain" response on the last three lessons. It seems reasonable that students became less impressed with the speed of the computer's response after the first few lessons.

5. Most people disagreed with the statement that "the mechanics of using the computer are very complicated." Nor surprisingly, the highest mean, (indicating a response closer to "uncertain"), occurred after the second lesson, which was the first time students used the computer without assistance.

6. Students were not usually frustrated by machine problems on the lessons. For lessons that came after the first lesson, about one-third of those answering the questionnaire indicated that they were uncertain, or that they had been frustrated by machine problems.

7. Responses to the final question, "this CAI lesson was more enjoyable than the previous one," closely paralleled responses to question three, in which students indicated whether or not they understood the material presented in the CAI lesson. Until the

last lesson, the mean response to this question was always between "uncertain" and "agree," but on the last lesson the mean response fell between "uncertain" and "disagree." Again, the reason for this decrease in attitude after the last lesson may reflect the quality of the lesson or the fact that the lesson occurred near the end of the school quarter, or it may be that students tend to tire of CAI after six lessons. Of these three possibilities, the last two seem more reasonable, because the mean response to the question on lesson 5 is lower than the mean response on previous lessons. That is, student attitudes may tend to decrease after the fourth lesson because the novelty has worn off, or because the end of the quarter is approaching.

The following recommendations are made for further study:

1. Replicate the present study to determine the generality of the present results.
2. Use the CAI units developed in this study as a supplement to lecture rather than as a replacement for lecture. This approach might be particularly useful with students who have previously been unsuccessful in studying mathematics, or students who have an inadequate background in mathematics.
3. The CAI units might be used by pairs of students rather than by individual students. Measures of achievement and attitudinal gains could be compared with gains made by students working individually on the terminals, or with students exposed to traditional instruction.

4. Duplicate the present study using a personality test in addition to the measures described in this study to determine if students who react positively to CAI can be distinguished on the basis of personality characteristics from those who react negatively to CAI.

5. Duplicate the present study with the lessons in a different order to determine whether the pattern of student response to the short attitudinal questionnaire is comparably changed. This procedure would help to determine whether a student were reacting to the content and format of a particular lesson, or whether he would have reacted similarly to almost any CAI lesson placed in that specific position within the sequence of lessons.

6. Duplicate the study with an improved set of CAI lessons, or with an expanded set of CAI lessons.

Recommendations for the classroom include the following:

1. The use of CAI as a part of a student's academic program does appear to be an effective means of improving his attitude toward CAI. In situations in which such improvement is an objective, this "hands-on" approach should definitely be considered.

2. Although students' attitudes toward mathematics did improve somewhat, these attitudes were still not very good, since the mean post-test attitudinal score was 53.47 in the control group and 61.75 in the CAI group. (A score of 60.00 represents a theoretically neutral attitude, with higher scores reflecting a higher attitude toward mathematics.) Attempts should be made to find

ways of improving these student attitudes.

3. Although the average gain in mathematics achievement was satisfactory, there are areas in which students still seem very weak, particularly in arithmetic that involves fractions. Students who cannot carry out basic arithmetic operations should be identified and helped through supplementary materials.

4. Computer-assisted instruction does appear to be a viable instructional strategy. Instructors should consider using CAI when it is appropriate for their educational objectives.

## SUMMARY

The objectives of this study were to develop six computer-assisted instruction lessons, and to evaluate the effectiveness of these lessons in teaching mathematics, in changing student attitudes toward CAI, and in changing student attitudes toward mathematics. In addition, the relationships among the major variables in the study were investigated, and a short assessment of attitudes was made after each of the six CAI units.

An experimental group and a control group, each consisting of 16 elementary education majors and 16 students not majoring in elementary education, were randomly selected from the 243 students enrolled in math 190 during winter quarter of 1972. Students in the control group received traditional instruction; students in the experimental group received the same instruction as the control group except that the experimental group was asked to miss class on six specified days. For each of these class periods, the experimental students were exposed to a CAI unit covering the same topic as the classroom lecture that was missed. All students were given a pre-test and a post-test measuring attitude toward CAI, attitude toward mathematics, and achievement in the mathematical content of the CAI units. Furthermore, a randomly-selected half of the experimental group was instructed to complete a short attitudinal questionnaire after each of the CAI units. The questionnaire measuring attitude toward CAI was a modified form of a questionnaire previously developed by Brown (68), and the questionnaire measuring attitude



toward mathematics was developed by Aiken and Dreger in (1), but the mathematics achievement measures and the short attitudinal questionnaire were developed specifically for this investigation.

The CAI units written for this study are in the form of computerized programmed instruction lessons dealing with topics that are a part of the curriculum of math 190 at Iowa State University. These units were written in CPS (Conversational Programming System), a time-sharing system that allows each student to interact with the computer by means of a typewriter terminal. The mean completion time for the first lesson was 42 minutes, but on subsequent lessons the mean completion time varied from 17 minutes to 26 minutes.

The major hypotheses of the study and the results of testing these hypotheses may be stated as follows:

Null hypothesis 1: There is no significant change in the control group's attitude toward CAI as a result of taking math 190. Since the calculated t statistic is a non-significant 1.73, this hypothesis cannot be rejected at the .05 level.

Null hypothesis 2: There is no significant change in the CAI group's attitude toward CAI as a result of taking math 190. For this hypothesis, the calculated t value is 7.19, which is significant beyond the .01 level.

Null hypothesis 3: There is no significant difference in the control group's attitude toward CAI and the CAI group's attitude toward CAI when initial pre-test differences have been controlled.

The F value for this test is a highly significant 27.89, so the third null hypothesis is rejected at both the .05 level and the .01 level.

Null hypothesis 4: There is no significant change in the control group's attitude toward mathematics as a result of taking math 190. Because the t statistic for this hypothesis, 2.10, falls slightly above significance level, this hypothesis is rejected at the .05 level.

Null hypothesis 5: There is no significant change in the CAI group's attitude toward mathematics as a result of taking math 190. The calculated t value in this case is a non-significant 1.72, so null hypothesis 5 cannot be rejected at the .05 level.

Null hypothesis 6: There is no significant difference in the control group's attitude toward mathematics and the CAI group's attitude toward mathematics when initial pre-test differences have been controlled. The F value for this hypothesis is a non-significant .09, so this hypothesis cannot be rejected at the .05 level.

Null hypothesis 7: There is no significant change in the control group's achievement in mathematics as a result of taking math 190. Since the t statistic for this hypothesis is a highly significant 7.38, null hypothesis 7 is rejected at the .05 level and at the .01 level.

Null hypothesis 8: There is no significant change in the CAI group's achievement in mathematics as a result of taking math

190. For this hypothesis, the calculated t statistic is 13.57, so this hypothesis is rejected at both the .05 level and at the .01 level.

Null hypothesis 9: There is no significant difference in the control group's achievement in mathematics and the CAI group's achievement in mathematics when initial pre-test differences have been controlled. In this case the appropriate F value is a non-significant 1.24, so this null hypothesis cannot be rejected at the .05 level.

To fulfill the secondary objectives of the study, correlation coefficients between all possible pairs of the six variables used in this study were calculated, and may be found in Tables 5 and 6, pages 52 and 53 of this paper. Also, the mean response to each question in the short attitudinal questionnaire was calculated for each CAI unit, and these results are printed in Figures 1-7, pp. 56-62 of this paper. An analysis of covariance procedure controlling on pre-test attitude toward CAI scores revealed no significant differences in post-test CAI attitudinal scores between students who completed these short attitudinal forms and students who were not given the forms.

The following conclusions may be drawn from this study:

1. Students' attitudes toward CAI did improve significantly as a result of using the CAI units.
2. Among students who did not use the CAI units, attitudes toward CAI remained relatively stable throughout the quarter.

There was no significant change in these students' attitudes toward CAI, and the correlation between pre-test CAI attitude and post-test CAI attitude was a highly significant .675 for these students.

3. In this study, CAI was not more effective than traditional instruction in improving a student's attitude toward mathematics. Attitudes toward mathematics improved in both the CAI group and the control group, but this change reached significance level only in the control group. However, the analysis of covariance procedure controlling on pre-test scores of attitude toward math revealed no significant differences between the two groups in post-test scores of attitude toward math.

4. Computer-assisted instruction and traditional instruction were both effective in producing achievement gains in students. Students in both the CAI group and the control group improved significantly in achievement, and there was no significant difference between the achievement scores of the two groups. The CAI students, however, spent less time in their instruction than did the traditionally instructed students.

5. Attitude toward CAI was unrelated to either attitude toward mathematics or achievement in mathematics, since the correlation coefficients between each variable concerning CAI attitude and each variable concerning mathematics attitude fell below significance level. On the other hand, there was a high degree of relationship between pre-test attitude toward mathematics and post-test attitude toward mathematics in the CAI group

( $r=.926$ ) and in the control group ( $r=.904$ ). There was a positive but moderate correlation between pre-test math achievement and pre-test math attitude in both the CAI group ( $r=.574$ ) and the control group ( $r=.512$ ), and between post-test mathematics achievement and post-test mathematics attitude in the CAI group ( $r=.515$ ). The correlation coefficient between post-test math attitude and post-test math achievement was lower in the control group ( $r=.314$ ). Another discrepancy between groups is the correlation coefficient between pre-test achievement in mathematics and post-test achievement in mathematics, since the correlation was  $.614$  in the CAI group, but only  $.229$  in the control group.

6. Completing the short attitudinal questionnaires did not significantly affect the students' response to the major questionnaire measuring attitude toward CAI.

7. Responses to the short attitudinal questionnaires revealed that students generally agreed that the computer acted like a person talking to them, that they understood the material presented in the CAI lessons, and that they enjoyed the current lesson more than the previous lesson. Their responses also indicated that they were not afraid they would damage the machine, that the computer was not slow in responding to their answers, that the mechanics of using the computer are not very complicated, and that they had not been frustrated by machine problems in working the lessons.

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## APPENDIX A: CAI LESSON TOPICS

- I. CAI unit 1
  - A. Converting base 12 numerals to base 10 numerals
  - B. Counting in a base 12 numeration system
  - C. Converting base 10 numerals to base 12 numerals
  - D. Working with numerals of the form  $23.4$ (twelve)
  - E. Review of multiplying and adding fractions
- II. CAI unit 2
  - A. The concept of arbitrary operations
  - B. Commutative property of arbitrary operations
  - C. Properties of subtraction
- III. CAI unit 3
  - A. Properties of division
  - B. Formal definition of division
  - C. Quotients involving a zero
- IV. CAI unit 4
  - A. Definition of "divisible"
  - B. Tests for divisibility by 2, 5, 4, 3, 9, 6, and 10
  - C. Generality of the divisibility tests
- V. CAI unit 5
  - A. Definition of "prime"
  - B. Review of square roots
  - C. Method of determining whether a number is prime

## VI. CAI unit 6

- A. Definition of "least common multiple"
- B. Three methods of finding the least common multiple
- C. Using the least common multiple in adding fractions

APPENDIX B: PRE-TEST QUESTIONNAIRE  
MEASURING ATTITUDE TOWARD CAI

Read each statement and decide which response most correctly describes your attitude toward the statement. Then mark the number corresponding to this response on the answer sheet only. (Please do not mark this questionnaire.)

1. While taking computer-assisted instruction I would feel challenged to do my best work.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

2. While taking computer-assisted instruction I would be concerned that I might not be understanding the material.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

3. While taking computer-assisted instruction I would feel isolated and alone.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

4. I would feel uncertain as to my performance in the programmed instruction relative to the performance of others.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

5. While taking computer-assisted instruction I would find myself just trying to get through the material rather than trying to learn.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

6. Computer-assisted instruction should not be used in any form in the elementary school.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

7. Computer-assisted instruction could be used effectively in many college classes.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

8. In a situation where I am trying to learn something, it is important to me to know where I stand relative to others.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

9. Computer-assisted instruction would make this course more interesting.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

10. While taking computer-assisted instruction I would be more involved in running the machine than in understanding the material.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

11. I feel I could work at my own pace with computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

12. Computer-assisted instruction makes the learning too mechanical.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

13. I would feel as if I had a private tutor while on computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

14. While taking computer-assisted instruction I would be aware of efforts to suit the material specifically to me.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

15. While taking computer-assisted instruction I would find it difficult to concentrate on the course material because of the hardware.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

16. Computer-assisted instruction would be too confusing for grade school children.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

17. Computer-assisted instruction is an inefficient use of the student's time.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

18. While on computer-assisted instruction I would encounter mechanical malfunctions.

1=all the time      2=most of the time      3=some of the time  
4=seldom      5=never

19. Computer-assisted instruction would make it possible for me to learn more quickly than traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

20. I would feel frustrated by the computer-assisted instruction situation.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

21. The computer-assisted instruction approach is inflexible.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

22. Even otherwise interesting material would be boring when presented by computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

23. In view of the effort I put into it, I would be satisfied with what I had learned while using computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

24. In view of the amount I would learn, I would say computer-assisted instruction is superior to traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

25. With a course such as the one I am taking, I would prefer computer-assisted instruction to traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

26. I am not in favor of computer-assisted instruction because it is just another step toward depersonalized instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

27. Computer-assisted instruction is too fast.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

28. Typing experience is necessary in order to perform satisfactorily on computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

29. Computer-assisted instruction is boring.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

APPENDIX C: POST-TEST QUESTIONNAIRE  
MEASURING ATTITUDE TOWARD CAI



Read each statement and decide which response most correctly describes your attitude toward the statement. Then mark the number corresponding to this response on the answer sheet only. (Please do not mark this questionnaire.)

1. While taking computer-assisted instruction I felt challenged to do my best work.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

2. While taking computer-assisted instruction I was concerned that I might not be understanding the material.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

3. While taking computer-assisted instruction I felt isolated and alone.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

4. I felt uncertain as to my performance in the programmed instruction relative to the performance of others.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

5. While taking computer-assisted instruction I found myself just trying to get through the material rather than trying to learn.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

6. Computer-assisted instruction should not be used in any form in the elementary school.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

7. Computer-assisted instruction could be used effectively in many college classes.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

8. In a situation where I am trying to learn something, it is important to me to know where I stand relative to others.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

9. Computer-assisted instruction made this course more interesting.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

10. While taking computer-assisted instruction I was more involved in running the machine than in understanding the material.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

11. I felt I could work at my own pace with computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

12. Computer-assisted instruction makes the learning too mechanical.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

13. I felt as if I had a private tutor while on computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

14. While taking computer-assisted instruction I was aware of efforts to suit the material specifically to me.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

15. While taking computer-assisted instruction I found it difficult to concentrate on the course material because of the hardware.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

16. Computer-assisted instruction would be too confusing for grade school children.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

17. Computer-assisted instruction is an inefficient use of the student's time.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

18. While on computer-assisted instruction I encountered mechanical malfunctions.

1=all the time      2=most of the time      3=some of the time  
4=seldom      5=never

19. Computer-assisted instruction made it possible for me to learn more quickly than traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

20. I felt frustrated by the computer-assisted instruction situation.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

21. The computer-assisted instruction approach is inflexible.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

22. Even otherwise interesting material would be boring when presented by computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

23. In view of the effort I put into it, I was satisfied with what I learned while using computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

24. In view of the amount I learned, I would say computer-assisted instruction is superior to traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

25. With a course such as the one I am taking, I would prefer computer-assisted instruction to traditional instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

26. I am not in favor of computer-assisted instruction because it  
it just another step toward depersonalized instruction.

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4=agree      5=strongly agree

27. Computer-assisted instruction is too fast.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

28. Typing experience is necessary in order to perform easily on  
computer-assisted instruction.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

29. Computer-assisted instruction is boring.

1=strongly disagree      2=disagree      3=uncertain  
4=agree      5=strongly agree

APPENDIX D: PRE-TEST MATHEMATICS ACHIEVEMENT MEASURE

Answer the following questions on this paper.

1. Using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e to write numerals in the base 12 number system, what is the base 12 numeral immediately following after  $59_{12}$ ?
2. Write the base 10 numeral for  $2t7_{12}$ .
3. Write  $21.34_6$  as a base 10 numeral.
4. Write  $268_{10}$  as a base 12 numeral.
5. Consider the operation  $*$  on the set of natural numbers where  $a*b = (a + b) - 2$ .
  - a) Find the numeric value of  $8*3$ .
  - b) Explain why the operation  $*$  is (or is not) commutative on the set of natural numbers.
6. State the commutative property of addition.
7. State a rule that can be used to tell whether a number is divisible by 5 without actually dividing the number by 5.
8. State a rule that can be used to tell whether a number is divisible by 4 without actually dividing the number by 4.
9. Consider the following statement: a number is divisible by  $n*m$  if and only if it is divisible by  $n$  and by  $m$ . If this statement is true for all whole numbers  $n$  and  $m$ , then write "true;" otherwise, find a pair of whole numbers (a value for  $n$  and a value for  $m$ ) for which the statement is false.
10. Which pair of whole numbers does  $\sqrt{78}$  lie between?
11. If 143 is prime, write "prime;" if not, name a number that divides 143 (other than 1 and 143).
12. The largest prime number that is less than or equal to  $\sqrt{200}$  is \_\_\_\_\_.
13. Use the set of all multiples of 8 and the set of all multiples of 12 to find the least common multiple of 8 and 12. Show your work.
14. If  $n*m = 400$  and the greatest common divisor of  $n$  and  $m$  is 2, what is the least common multiple of  $n$  and  $m$ ?

15. Find the least common multiple of 18 and 24.
16. To see if 101 is a prime number, it would be necessary to try to divide 101 by exactly four numbers. Name those four numbers.
17. Add  $1/18 + 5/24$ .

For each of the following questions, circle the letter corresponding to the correct response.

18. The statement  $18 - 0 = 0 - 18$
- is false.
  - is true because of the zero property of subtraction.
  - is true because of the commutative property of subtraction.
  - is true because of the associative property of subtraction.
19. The statement  $(6 + 15) \div 3 = (6 \div 3) + (15 \div 3)$
- is false.
  - is true because of the commutative property of division.
  - is true because of the associative property of division.
  - is true because of the right distributive property of division over addition.
20. The statement  $1 \div (3 + 5) = (1 \div 3) + (1 \div 5)$
- is false.
  - is true because of the commutative property of division.
  - is true because of the associative property of division.
  - is true because of the right distributive property of division over addition.
21. The statement  $(12 \div 4) \div 1 = 12 \div (4 \div 1)$
- is true because of the commutative property of division.
  - is true because of the associative property of division.
  - is true because of the distributive property of division.
  - is true, but not for any of the above reasons.
22. The expression  $0 \div 0$
- is equal to 1, because a number divided by itself is always 1.
  - is meaningless.
  - is equal to 0, since 0 divided by anything is always 0.
  - is equal to infinity.

23. To see that 864123 is divisible by 3 without actually dividing, one could notice that
- a) the number in the one's position is divisible by 3.
  - b) the number named by the last three digits (123) is divisible by 3
  - c) the sum of the digits is divisible by 3.
  - d) none of the above.



APPENDIX E: POST-TEST MATHEMATICS ACHIEVEMENT MEASURE

## Math 190

## Review Quiz

Circle the letter corresponding to the correct response:

1. The statement  $1 \div (4 + 5) = (1 \div 4) + (1 \div 5)$ 
  - a) is false.
  - b) is true because of the commutative property of division.
  - c) is true because of the associative property of division.
  - d) is true because of the right distributive property of division over addition.
  
2. The statement  $(16 \div 4) \div 1 = 16 \div (4 \div 1)$ 
  - a) is true because of the commutative property of division.
  - b) is true because of the associative property of division.
  - c) is true because of the right distributive property of division over addition.
  - d) is true, but not for any of the above reasons.
  
3. The expression  $0 \div 0$ 
  - a) is equal to 1.
  - b) is undefined.
  - c) is equal to 0.
  - d) is equal to infinity.
  
4. The statement  $(6 + 15) \div 3 = (6 \div 3) + (15 \div 3)$ 
  - a) is false.
  - b) is true because of the commutative property of division.
  - c) is true because of the associative property of division.
  - d) is true because of the right distributive property of division over addition.
  
5. The statement  $1 - 0 = 0 - 1$ 
  - a) is false.
  - b) is true because of the zero property of subtraction.
  - c) is true because of the commutative property of subtraction.
  - d) is true because of the associative property of subtraction.
  
6. To see that 864123 is divisible by 3 without actually dividing, one could notice that
  - a) the number in the one's position is divisible by 3.
  - b) the number named by the last three digits is divisible by 3.
  - c) the sum of the digits is divisible by 3.
  - d) none of the above.
  
7. Write  $23.32_4$  as a base 10 numeral.
  
8. Consider the operation  $*$  on the set of natural numbers where  $a*b = 5a+b$ 
  - a) Find the numeric value of  $8*3$

- b) Explain why the operation  $*$  is (or is not) commutative on the set of whole numbers.
9. State the commutative property of multiplication.
10. Use the number 679 to explain why the divisibility test for 2 works as it does.
11. State a rule that can be used to tell whether a number is divisible by 6 without actually dividing the number by 6.
12. Consider the following statement: a number is divisible by  $n \cdot m$  if and only if it is divisible by  $n$  and by  $m$ . If this statement is true for all natural numbers  $n$  and  $m$ , then write "true;" otherwise, find a pair of natural numbers (a value for  $n$  and a value for  $m$ ) for which the statement is false.
13. Name the pair of consecutive whole numbers that  $\sqrt{87}$  lies between.
14. If 247 is prime, write "prime." If not, name a number that divides 247 (other than 1 and 247).
15. The largest prime number that is less than or equal to  $\sqrt{180}$  is \_\_\_\_\_.
16. Using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e to write numerals in the base 12 numeration system, what is the base 12 numeral immediately following  $5t9_{12}$ ?
17. Write the base 10 numeral for  $e8_{12}$ .
18. Write  $136_{10}$  as a base 12 numeral.
19. Use the set of all multiples of 10 and the set of all multiples of 12 to find the least common multiple of 10 and 12. Show your work (on the answer sheet).
20. If  $n \cdot m = 1200$  and the greatest common divisor of  $n$  and  $m$  is 4 what is the least common multiple of  $n$  and  $m$ ?
21. Find the least common multiple of 24 and 36.
22. Add  $1/24 + 5/36$ .
23. To see if 157 is a prime number, it would be necessary to try to divide 157 by exactly five numbers. Name those 5 numbers.

## APPENDIX F: PROGRAM LISTING

## Routines Common to All Units

Each of the CAI units consists of a main program and two or three segments called "procedures." Within each procedure are the following two sets of commands: a set of declaration and initialization statements, and a group of statements that process the student's response. Because the declaration and initialization statements, (statements 58-69), and the processing statements, (statements 70-98.1), are identical in each procedure, these statements are printed separately on the following page, and are omitted from the remaining program listings.

```

58.      DECLARE now LABEL , yes LABEL , no LABEL , dno LABEL , next LABEL , where LABEL ;
59.      DECLARE fine LABEL , nbik LABEL , news CHAR(50) VAR;
60.      fine=ok;
61.      nbik=lpvt;
62.      now=ahora;
63.      yes=ya;
64.      dno=dnein;
65.      no=nein;
66.      nrt=0;
67.      where=here;
68.      lr=1;
69.      lw=0;

70.  ahora: PUT LIST('Reply:');
71.      READ INTO(reply) ;
71.1     e=reply;
72.      reply=upcase(reply);
73.      IF index(reply,' ') = 0 THEN GO TO nbik;
74.      len=length(reply);
75.      IF len <= 2 THEN GO TO nbik;
76.      news=substr(reply,1,1);
77.      cha: DO M=2 TO len;
78.          IF substr(reply,M,1)=' ' THEN GO TO fine;
79.          news=news||substr(reply,M,1);
80.      ok:  END cha;
81.          reply=news;
82.      lpvt: DO K=1 TO lr;
83.          IF index(reply,ans(K)) > 0 THEN GO TO yes;
84.      END lpvt;
85.      lwze: IF lw=0 THEN GO TO no;
86.      lpno: DO K=1 TO lw;
87.          KK=K;
88.          IF index(reply,wans(K)) > 0 THEN GO TO dno;
89.      END lpno;
90.      GO TO no;
91.      ya:  PUT LIST(cor);
92.          nrt=nrt+1;
93.      GO TO where;
94.      nein: PUT LIST(unrc);
95.          GO TO where;
96.      dnein: PUT LIST(diag(KK));
97.      here: lw=0;
98.          lr=1;
98.1     GO TO next;

```

CAI UNIT 1

```

1. DECLARE TW1 ENTRY EXT;
1.1 CALL TW1;
2. DECLARE TW2 ENTRY EXT;
2.1 PUT LIST('Part 2: In case of malfunction, req 2 thru...');
2.2 PUT LIST('Which base twelve numeral would follow e?');
2.3 CALL TW2;
3. DECLARE TW3 ENTRY EXT;
3.1 PUT LIST('Part 3: In case of malfunction, req 3 thru...');
3.2 PUT LIST('To solve 236(ten)=____(twelve), we could proceed as follows:');
3.3 PUT LIST('236/144=1, remainder 92; 92/12=7, remainder 8; 8/1=8, remainder 0.');
```

3.4 PUT LIST('Thus 236(ten)=178(twelve). The 1,7, and 8 were the quotients in each of the steps above.');

3.5 PUT LIST('Convert 308(ten) to a base twelve numeral.');

3.6 CALL TW3;

4. END;

```

-- 55. TW1: PROCEDURE ;
56. DECLAR ans(2) CHAR(30) VAR, wans(5) CHAR(20) VAR, cor CHAR(254) VAR, dlag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57. DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

69.1 PUT LIST('In a base n number system, the positions represented are 1,n,nxn,nxnxn,nxnxxn, and so on.');
```

69.15 PUT LIST('In a base 12 number system, therefore, the positions represented are 1, 12, 12x12, \_\_\_\_\_, and so on.');

```

69.2 PUT LIST('(Type the number that belongs in the blank--you may use the letter x to denote multiplication.)');
69.25 ans(1)='12X12X12X12';
69.3 ans(2)='TWELVEXTWELVEXTWELVEXTWELVE';
69.35 wans(1)='12X12X12';
69.4 wans(2)='12CUBED';
69.45 wans(3)='TWELVECUBED';
69.5 wans(4)='TWELVEXTWELVEXTWELVE';
69.52 wans(5)='1728';
69.55 cor='No, 12x12x12 is the correct answer.';
69.6 dlag(1)='Yes, 12x12x12 is correct.';
69.65 dlag(2)=dlag(1);
69.7 dlag(3)=dlag(1);
69.75 dlag(4)=dlag(1);
69.77 dlag(5)=dlag(1);
69.8 unrc=cor;
69.85 lr=2;
69.9 lw=5;
69.93 next=k2;
69.95 GO TO now;

-- 100. k2: nrt=0;
101. PUT LIST('To change a base 10 numeral to a base 12 numeral, just multiply the place value of each position');
102. PUT LIST('(1, 12, 144, 1728,...) by the numeral in that position; then add each of the products. For example');
103. PUT LIST('in the numeral 203(twelve), the place values-reading from left to right- are 144, 12, and 1. Thus');
104. PUT LIST('203(twelve)=(2x144)+(0x12)+(3x1)=288+0+3=291(ten). What is the base 10 representation of 37(twelve)?');

105. ans(1)='43';
105.1 wans(1)='+';
105.2 d='Please record only the final form of the answer__';
105.3 dlag(1)=d||' 43(ten)';
105.4 lw=1;
106. cor='Fine. 37(twelve)=43(ten).';
107. unrc='Not quite. 37(twelve)=(3x12)+(7x1)=36+7=43(ten).';
108. next=k3;
109. GO TO now;

```

```

110. k3: PUT LIST('Now change 315(twelve) to a base 10 numeral.');
```

ans(1)='449';  
diag(1)=d||' 449(ten)';  
lw=1;  
cor='Right, 315(twelve)=449(ten).';  
unrc='No, in 315(twelve) the positions represented are 144, 12, and 1, so 315(twelve)=3x144+1x12+5x1=432+12+5=449  
(ten)';

```

114.     next=k4;
115.     GO TO now;
116. k4: next=k6;
117.     IF nrt=2 THEN GO TO next;
118.     PUT LIST('Convert 147(twelve) to a base 10 numeral.');
```

ans(1)='199';  
diag(1)=d||' 199(ten)';  
lw=1;  
cor='Good, 147(twelve)=199(ten)';  
unrc='No, 147(twelve)=1x144+4x12+7x1=144+48+7=199(ten).';  
next=k5;  
GO TO now;

```

124. k5: next=k6;
125.     IF nrt=2 THEN GO TO next;
126.     PUT LIST('Try one more: convert 270(twelve) to a base 10 numeral.');
```

ans(1)='372';  
diag(1)=d||' 372(ten)';  
lw=1;  
cor='Fine, 270(twelve)=372(ten)';  
unrc='Not exactly: In 270(twelve) the positions represented are 144,12,and 1, so 270(twelve)=2x144+7x12+0x1=372(t  
en)';

```

130.     GO TO now;
131. k6: PUT LIST('Counting in the base twelve system is a little strange, but it is easier if you think');
```

PUT LIST('about a base 12 abacus. On a base ten abacus, we replace ten beads on any wire with a single');  
PUT LIST('bead on the wire to the left of the original wire; this means that there are never any more');  
PUT LIST('than nine beads on any wire, so the base ten system needs numerals to represent the whole');  
PUT LIST('numbers from 0 through 9. What is the largest number of beads that can be on any wire of a');  
PUT LIST('base 12 abacus?');

```

137. ans(1)='11';
138. ans(2)='ELEVEN';
139. lr=2;
140. cor='Yes, the answer is eleven.';
141. a='No. On a base 12 abacus, 12 beads on a wire are replaced by 1 bead on the next wire,';
142. b='so there can be no more than 11 beads on any wire.';
143. unrc=a||b;
144. next=k7;
145. GO TO now;
146. k7: PUT LIST('Thus a base twelve system needs numerals to represent the whole numbers from 0 through eleven. The');
```

PUT LIST('base 12 numerals are the same as base 10 numerals from 0 through 9. Think about having 9 beads on  
the');  
PUT LIST('one''s wire of an abacus, and then adding a single bead to the one''s wire. We now have ten beads on  
the');  
PUT LIST('one''s wire, so we need a numeral to represent this arrangement. (We can't use 10, since 10(twelve)  
must');  
PUT LIST('mean 1x12 + 0x1 =12(ten).) Although almost any symbol could be used, we will use the letter t to  
represent');  
PUT LIST('ten just because that symbol is easy to remember. Thus counting in the base twelve system goes as  
follows:');  
PUT LIST('1,2,3,4,5,6,7,8,9,t...');

PUT LIST('If you had to guess, which symbol do you think would be used for the number that follows t?');



```

153.5      ans(1)='E';
153.6      wans(1)='11';
154.        lw=1;
155.        cor='Yes, e will be used to represent eleven in the base twelve system.';
156.        diag(1)='No, 11 can't be used, since 11(twelve)=1x12+1x1=13(ten). Instead, we will use the letter e.';
157.        unrc='Very interesting.';
158.        next=k8;
159.        nrt=0;
160.        GO TO now;
161.        next=k9;
162.        k8:  IF nrt=1 THEN GO TO next;
163.           IF index(reply,'11')>0 THEN GO TO next;
164.           unrc='Actually, though, we will use the letter e as the symbol for eleven in the base 12 system.';
165.           cor=reply||' could be used, but we will use the letter e instead.';
166.           IF length(reply)=1 THEN PUT LIST(cor); ELSE PUT LIST(unrc);
167.        k9:  END ;

-- 55.      TW2:  PROCEDURE ;
56.         DECLARE ans(2) CHAR(30) VAR, wans(5) CHAR(20) VAR, cor CHAR(254) VAR, diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57.         DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

-- 69.1      ans(1)='10';
69.2      wans(1)='TEN';
69.3      lw=1;
69.4      cor='Right.';
69.5      unrc='No, the answer should be 10(twelve).';
69.6      diag(1)='No, "ten" means 10(ten); you want 10(twelve), which is read "one-zero".';
69.7      next=k10;

-- 200.     k10:  next=k11;
201.         IF nrt=1 THEN GO TO next;
202.         PUT LIST('Think of having eleven beads on the one's wire of an abacus and then adding one more bead to');
203.         PUT LIST('the one's wire. That makes twelve beads, so we replace all twelve by a single bead on the next');
204.         PUT LIST('wire. Thus we have one bead on the twelve's wire, and nothing on the one's wire. But the numeral');
205.         PUT LIST('that represents this arrangement must be one-zero--that is, 10(twelve).');
206.     k11:  PUT LIST('Thus our counting proceeds as follows:');
207.         PUT LIST(' 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e');
208.         PUT LIST('10,11,12,13,14,15,16,17,18,19,...');
209.         PUT LIST('Which numeral will succeed 19(twelve)?');
210.         ans(1)='1T';
211.         a='After this come the numerals 1e,20,21,22,...';
212.         cor='Good, 1t(twelve) follows 19(twelve). '||a;
213.         unrc='Not exactly: the answer is 1t(twelve).';
214.         next=k12;
215.         GO TO now;
216.     k12:  next=k13;
217.         IF index(reply,'1T')>0 THEN GO TO next;
218.         PUT LIST('You see 19(twelve) means 1-twelve + 9-one's. If you add one to that number, the result will');
219.         PUT LIST('be 1-twelve + ten-one's. Since t=ten, this number is represented by 1t(twelve).');
220.         PUT LIST(a);
221.     k13:  next=k16;
222.         IF nrt>1 THEN GO TO next;
223.         PUT LIST('After 22(twelve) come 23,24,25,26,27,28,29,2t,and_____');
224.         ans(1)='2E';
225.         a='2e is followed by 30,31,32,33,34,35,36,37,38,39,3t,3e,and 40.';
226.         cor='Fine, 2e(twelve) is correct. '||a;
227.         unrc='The answer is 2e(twelve).';

```

```

228.      next=k14;
229.      GO TO now;
230.      k14:  next=k14A;
231.      IF index(reply,'2E')>0 THEN GO TO next;
232.      PUT LIST('2t(twelve) means 2-twelves + ten-ones; if you add 1 to this mess, you will get 2-twelves +eleven
ones,');
233.      PUT LIST('which is represented as 2e(twelve).');
234.      PUT LIST(a);
234.5    k14A: next=k16;
234.6    IF nrt>1 THEN GO TO next;
235.      PUT LIST('if we continue counting in this way, we will reach 49, 4t, 4e, _____(twelve).');
236.      ans(1)='50';
237.      cor='Very good, 50(twelve) comes after 4e(twelve).';
238.      next=k15;
239.      GO TO now;
240.      k15:  next=k16;
241.      IF nrt>1 THEN GO TO next;
242.      PUT LIST('Since e=eleven, 4e(twelve) means (4x12) + (11x1). If we add 1 to this number, we then have');
243.      PUT LIST('(4x12) + (12x1) = 5x12 =5-twelves + 0 one''s. But this is represented as 50(twelve).');
244.      k16:  PUT LIST('Which numeral comes after 3e(twelve)?');
245.      ans(1)='3E0';
246.      cor='Fine, 3e0(twelve) is correct.';
247.      unrc='No, the answer is 3e0.';
248.      next=k17;
249.      GO TO now;
250.      k17:  next=k18;
251.      IF index(reply,'3E0')>0 THEN GO TO next;
252.      PUT LIST('3e means 3x144 + 10x12 + 11x1. If we add 1 more, this makes 3x144 + 10x12 + 12x1, which');
253.      PUT LIST('is the same as 3x144 + 11x12 + 0x1=3e0(twelve).');
254.      k18:  PUT LIST('Try one more: which numeral comes after 3ee(twelve)?');
255.      ans(1)='400';
256.      a='400(twelve) is correct.';
257.      cor='Good, '|a;
258.      unrc='Wrong! '|a;
259.      next=k19;
260.      GO TO now;
261.      k19:  next=k20;
262.      IF index(reply,'400')>0 THEN GO TO next;
263.      PUT LIST('3ee means 3x144 + 11x12 + 11x1. Adding 1 to this number, we get 3x144 + 11x12 + 12x1, which');
264.      PUT LIST('is equal to 3x144 + 12x12, which is really 4x144. Thus the answer is 400(twelve).');
265.      k20:  PUT LIST('To change a base ten numeral to a base twelve numeral, follow the general rule for converting');
266.      PUT LIST('a base 10 numeral to a base n numeral: divide the base ten numeral by the highest power of 12 that');
267.      PUT LIST('is less than or equal to the base ten numeral; then divide the remainder by the next lower power of');
268.      PUT LIST('12; continue this process for each lower power of 12, including the number 1. The only thing
different');
269.      PUT LIST('about this conversion is that a quotient of 10 or 11 (should you get one) must be changed to t or e. ');
270.      PUT LIST('Incidentally, do you know why 1 can be considered a power of 12? (Answer yes or no.)');
271.      ans(1)='YES';
272.      cor='Fine, then you can answer the following question: ';
273.      a='1 can be considered a power of 12 because twelve with an exponent zero is equal to 1.';
274.      unrc=a;
275.      next=k21;
276.      GO TO now;
277.      k21:  next=k22;
278.      IF index(reply,'NO')>0 THEN GO TO next;
278.5    PUT LIST('Twelve with an exponent _____ is equal to 1. ');
279.      ans(1)='0';

```

```

280.      ans(2)='ZERO';
281.      lr=2;
282.      cor='Yes, '||a;
283.      unrc='No, '||a;
284.      GO TO now;
285. k22:  END ;

- 55.    TW3:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(30) VAR, wans(5) CHAR(20) VAR, cor CHAR(254) VAR, diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57.      DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

- 69.1   ans(1)='218';
69.2   cor='Good, 308(ten)=218(twelve).';
69.3   next=k23;
69.4   unrc='No, 218(twelve) is the answer.';

- 300.   k23:  next=k24;
301.      IF nrt=1 THEN GO TO next;
302.      PUT LIST('The largest power of 12 that is less than or equal to 308 is 144, so the division process');
303.      PUT LIST('starts as follows: 308/144=2, remainder 20. ');
304.      PUT LIST('Now divide the remainder by the next lower power of 12, which is 12 itself: 20/12=1 remainder 8 ');
305.      PUT LIST('Divide by the next lower power of 12: 8/1=8 remainder 0. Thus 308(ten)=218(twelve). The ');
306.      PUT LIST('digits 2, 1, and 8 were the quotients in each of the three divisions. ');
307. k24:  PUT LIST('Convert 135(ten) to a base 12 numeral. ');
308.      ans(1)='E3';
309.      cor='Right, 135(ten)=e3(twelve).';
310.      unrc='No, the division would proceed as follows: ';
311.      next=k25;
312.      GO TO now;
313. k25:  next=k26;
314.      IF Index(reply, 'E3')>0 THEN GO TO next;
315.      PUT LIST('135/12=11, remainder 3; 3/1=3, remainder 0. Thus the answer is e3(twelve), since e represents ');
316.      PUT LIST('eleven in the base 12 system. A frequent mistake is to represent the numeral as 113(twelve), but ');
317.      PUT LIST('113(twelve) would have to mean 1x144 + 1x12 + 3x1 =159(ten), which is incorrect. ');
318. k26:  next=k27;
319.      IF nrt>=2 THEN GO TO next;
320.      PUT LIST('Try one more. Convert 264(ten) to a base twelve numeral. ');
321.      ans(1)='1T0';
322.      cor='Correct, 264(ten)=1t0(twelve).';
323.      unrc='No: 264/144=1 remainder 120; 120/12=10 remainder 0; 0/1=0 remainder 0. Hence 264(ten)=1t0(twelve).';
324.      GO TO now;
325. k27:  PUT LIST('Now let''s consider numbers like 23.4(twelve). In a base 12 system, the value of each position ');
326.      PUT LIST('should be 1/12 times the value of the position to the left. From left to right, then, the place
values ');
327.      PUT LIST('of 23.4 are 12, 1, and _____. (Please answer with a number, not a word. Guess if you don''t know. ');
328.      ans(1)='1/12';
329.      cor='Right, 1/12 is the place value of the next position. ';
330.      unrc='No, the answer is 1/12 times the place value on the left, which equals 1/12 times 1, or 1/12. ';
331.      next=k28;
332.      GO TO now;
333. k28:  PUT LIST('What, then, is the base ten representation of 23.4(twelve)? ');
334.      ans(1)='274/12';
335.      ans(2)='271/3';
336.      lr=2;
337.      cor='Very good, 23.4(twelve)=27 1/3. ';
338.      unrc='Not quite: 23.4(twelve)=(2x12)+(3x1)+(4x 1/12)=24+3+ 4/12, or 27 1/3';
339.      next=k29;
340.      GO TO now;

```

```

341. k29: nrt=0;
342. PUT LIST('In general, then, the positions represented in a base n numeral are as follows');
343. PUT LIST('...nxxnxxn, nxxn, n, 1, 1/n, 1/(nxn), 1/(nxxn), 1/(nxxnxxn),...');
344. PUT LIST('For example, .234(ten) really means  $2x(1/10) + 3x(1/(10x10)) + 4x(1/(10x10x10))$ ; on the other hand,');
345. PUT LIST('.234(five) means  $2x(1/5) + 3x(1/(5x5)) + 4x(1/(5x5x5)) = 2/5 + 3/25 + 4/125 = 50/125 + 15/125 + 4/125$ ');
346. PUT LIST('69/125, so .234(five)=69/125 (ten).');
347. PUT LIST('What is the base ten representation of 3t.29(twelve)?');
348. ans(1)='4633/144';
349. ans(2)='4611/48';
350. lr=2;
351. cor='Very good, 3t.29(twelve)= 46 11/48 (ten).';
352. unrc='No, 3t.29(twelve)=(3x12)+(10x1)+(2x 1/12)+(9x 1/144)=36+10+ 24/144 + 9/144=46 33/144 = 46 11/48 (ten).';
353. next=k30;
354. GO TO now;
355. k30: PUT LIST('Perform the following conversion: 142.51(six)=_____(ten).');
356. ans(1)='6231/36';
357. cor='Good, 142.51(six)=62 31/36(ten).';
358. unrc='Not quite: 142.51(six)=(1x36)+(4x6)+(2x1)+(5x 1/6)+(1x 1/36)=36+24+2+30/36 +1/36=62 31/36 (ten).';
359. next=k31;
360. GO TO now;
361. k31: next=k32;
362. IF nrt>0 THEN GO TO next;
363. PUT LIST('Remember that to multiply fractions, you simply multiply the numerators and multiply the denominators.'
);
364. PUT LIST('For instance,  $2/3 \times 5/7 = (2x5)/(3x7) = 10/21$ . To add fractions, however, you must first change to a');
365. PUT LIST('common denominator and then add the numerators. Thus  $5/6 + 1/36 = (5x6)/(6x6) + 1/36 = 30/36 + 1/36 =$ 
31/36. ');
366. PUT LIST('You get the 30/36 from 5/6 by multiplying the numerator and denominator of 5/6 by 6. This is legal,
because');
367. PUT LIST('it is equivalent to multiplying by 6/6, which is 1, and multiplying by 1 doesn't change the number's
value. ');
368. k32: PUT LIST('Try one more: What is the base ten representation of 32.13(four)?');
369. ans(1)='147/16';
370. cor='Good, 32.13(four)=14 7/16 (ten).';
371. unrc='No, 32.13(four)=(3x4)+(2x1)+(1x1/4)+(3x 1/16)=12+2+ 4/16 + 3/16=14 7/16 (ten).';
372. next=k33;
373. GO TO now;
374. k33: PUT LIST('End of lesson--you may logout. ');
375. END ;

```

CAI UNIT 2

```

1.      DECLARE C1 ENTRY EXT;
1.1     CALL C1;
2.      PUT LIST('
thru...');
2.1     DECLARE COM2 ENTRY EXT;
2.2     PUT LIST('The commutative property of addition on the whole numbers says that a+b=b+a for all whole numbers');
2.3     PUT LIST('a and b, and the commutative property of set union says that A∪B=B∪A for all sets A and B. Then if
an-1);
2.4     PUT LIST('arbitrary operation called * is commutative on the set of whole numbers, it must be true that_____
');
2.5     PUT LIST('for all whole numbers a and b. (Your answer should be an algebraic equation in a and b.)');
2.6     CALL COM2;
3.      DECLARE C3 ENTRY EXT;
3.1     PUT LIST('
part 3: In case of malfunction, req
3 thru...');
3.2     PUT LIST('Hence (9-6)-2 is not equal to 9-(6-2), so subtraction is not associative. ');
3.3     PUT LIST('We have said that the set of whole numbers, W, is closed under addition because whenever a and b are
in W, ');
3.4     PUT LIST('then the sum a+b is also in W. The same thing isn't true of subtraction, however, because both 2 and
9 are ');
3.5     PUT LIST('in W, but the difference_____is not in W. ');
3.6     CALL C3;
4.      END ;

55.     C1:  PROCEDURE ;
56.     DECLARE ans(2) CHAR(30) VAR, wans(5) CHAR(20) VAR, cor CHAR(254) VAR, diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57.     DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

69.1    PUT LIST('Of course you are familiar with the ordinary operations of addition, subtraction, multiplication, and
division, ');
69.2    PUT LIST('but there are many more operations that can be defined on our number system. Let's say, for example,
that each ');
69.3    PUT LIST('correct answer on page 1 of an exam is worth 2 points, and every correct answer on page 2 is worth 3
points. ');
69.4    PUT LIST('If a student has n correct answers on page 1 and m correct answers on page 2, his total score is
(2xn)+(3xm). ');
69.5    PUT LIST('This could be considered a new operation that we will call *. For any numbers n and m, then, n*m=2n+3m.
');
69.6    PUT LIST('For instance, 25*10=(2x25)+(3x10)=50+30=80. What is the numeric value of 8*5? (Please give the answer
as a ');
69.7    PUT LIST('numeral, not an expression. ');
69.8    ans(1)='31';
69.85   wans(1)='*';
69.87   d='Please give the final answer only. In this case, the answer is ';
69.89   cor='Good, 8*5=31.';
69.91   unrc='No, the answer is 31: n*m=(2xn)+(3xm), so 8*5=(2x8)+(3x5)=16+15=31.';
69.93   diag(1)=d||'31; 8*5=(2x8)+(3x5)=16+15=31.';
69.95   next=k2;

100.    k2:  PUT LIST('Try another one: What is 35*2? ');
101.    ans(1)='76';
102.    cor='Right, 35*2=76.';
103.    unrc='No, 76 is correct: n*m=(2xn)+(3xm), so 35*2=(2x35)+(3x2)=70+6=76. Just substitute n=35, m=2 in the formula
n*m';
104.    diag(1)=d||'76. Just substitute n=35, m=2 in the formula for n*m: 35*2=(2x35)+(3x2)=70+6=76.';
105.    next=k3;
106.    GO TO now;

```

```

107. k3: next=k4;
108. IF nrt=2 THEN GO TO next;
109. PUT LIST('Try one more of this kind. What is the value of 30*10?');
110. ans(1)=90';
111. cor='Right, 30*10=90.';
112. unrc='Not quite. n*m=(2xm)+(3xm), so using n=30 and m=10, we get 30*10=(2x30)+(3x10)=60+30=90; thus 30*10=90.';
113. diag(1)=d||'90; n*m=(2xn)+(3xm), so 30*10=(2x30)+(3x10)=90.';
114. GO TO now;
115. k4: PUT LIST('Let's try an easier operation. Define n$m=(n+m)/2 for all whole numbers n and m. Hence for any
whole');
116. PUT LIST('numbers n and m, the expression n$m is the arithmetic average of n and m. For example, 7$13=(7+13)/2=20
/2=10.');
```

```

117. PUT LIST('What is 15$11?');
118. ans(1)=13';
119. wans(1)='';
120. cor='Fine, 15$11=13.';
121. unrc='The answer should be 13; n$m=(n+m)/2, so 15$11=(15+11)/2=26/2=13.';
122. diag(1)=d||'13; 15$11=(15+11)/2=26/2=13.';
123. next=k5;
124. nrt=0;
125. GO TO now;
126. k5: next=k6;
127. IF nrt=1 THEN GO TO next;
128. PUT LIST('Find the value of 20$40.');
```

```

129. ans(1)=30';
130. cor='Very good, 20$40=30.';
131. unrc='No, 20$40=(20+40)/2=60/2=30.';
132. diag(1)=d||'30, 20$40=(20+40)/2=60/2=30.';
133. GO TO now;
134. k6: PUT LIST('We could define the operation # as follows: let n#m=nm-1 for all natural numbers n and m. For
instance,');
```

```

135. PUT LIST('9#8=(9x8)-1=72-1=71. For any pair of numbers, the "answer" is their product minus 1. What is the
value');
```

```

136. PUT LIST('of 4#10?');
137. ans(1)=39';
138. wans(1)='-';
139. cor='Good, 4#10=39.';
140. nrt=0;
141. unrc='Not exactly. If n#m=nm-1, then 4#10=(4x10)-1=40-1=39. Just substitute n=4, m=10 into the formula for n#m.';
142. diag(1)=d||'39. Since n#m=nm-1, therefore 4#10=(4x10)-1=40-1=39.';
143. next=k7;
144. GO TO now;
145. k7: next=k8;
146. IF nrt=1 THEN GO TO next;
147. PUT LIST('Find 11#5.');
```

```

148. ans(1)=54';
149. cor='Correct, 11#5=54.';
150. unrc='No, 11#5=(11x5)-1=55-1=54. Substitute n=11, m=5 in the equation n#m=nm-1.';
151. diag(1)=d||'54. You see, 11#5=(11x5)-1=55-1=54.';
152. GO TO now;
153. k8: PUT LIST('Since mathematicians sometimes have to work with exponents, the following operation is often useful:');
154. PUT LIST('let n**m=n with an exponent m (that is, n raised to the mth power, or the product of m factors of n).
For');
```

```

155. PUT LIST('instance, 2**3 means 2 to the third power, or 2x2x2, which is 8. Similarly, 5**2 means 5 to the second
power');
```

```

156. PUT LIST('or 5x5, which is 25. (SO 2**3=8 and 5**2=25.) What is the value of 3**4? (Type a numeral, not an
expression.));
```

```

157.      ans(1)='81';
158.      wans(1)='X';
159.      cor='Yes, 3**4=81.';
160.      diag(1)=d||'81: 3**4=3x3x3x3=81.';
161.      unrc='Wrong. 3**4 is 3 raised to the fourth power, which is 3x3x3x3, which is 81. Thus 3**4=81.';
162.      next=k9;
163.      nrt=0;
164.      GO TO now;
165.      k9:  next=k10;
166.           IF nrt=1 THEN GO TO next;
167.           PUT LIST('Find the value of 2**5.');
```

ans(1)='32';

```

169.      cor='Right, 2**5=2x2x2x2x2=32.';
170.      diag(1)=d||'32, since 2**5=2x2x2x2x2=32.';
171.      unrc='Not exactly. 2**5 means 2 raised to the fifth power, which is 2x2x2x2x2, or 32. Hence 2**5=32.';
172.      GO TO now;
173.      k10: next=k11;
174.           IF nrt>0 THEN GO TO next;
175.           PUT LIST('5**3 means 5 to the third power, or _____.(Type the numeric value, not an expression.)');
```

ans(1)='125';

```

177.      cor='OK, 5**3=125.';
178.      diag(1)=d||'125. 5**3=5x5x5=125.';
179.      unrc='I'm afraid not: 5 to the third power means 5x5x5, (5 used as a factor 3 times) which is 125. Thus
5**3=125.';
180.      GO TO now;
181.      k11:  END ;
```

55. COM2: PROCEDURE ;

56. DECLARE ans(2) CHAR(10) VAR, wans(5) CHAR(10) VAR, cor CHAR(200) VAR, diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;

57. DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

```

69.1      ans(1)='B*A';
69.2      cor='Right, if * is commutative, then a*b=b*a.';
69.3      unrc='No, if the operation * is commutative on the whole numbers, then a*b=b*a for all whole numbers a and b.';
69.4      next=k12;
```

```

200.      k12:  PUT LIST('If we use the particular operation * described in problem 1 (that is, n*m=(2xn)+(3xm) for any');
201.           PUT LIST('whole numbers n and m), then 25*10=80. What is the numeric value of 10*25?');
202.           ans(1)='95';
203.           wans(1)='*';
204.           lw=1;
205.           cor='Yes, 10*25=95.';
206.           d='Please just type the final answer.';
207.           unrc='Not quite. If n*m=(2xn)+(3xm), then 10*25=(2x10)+(3x25)=20+75=95. Just substitute n=10, m=25 in the
formula n*m.';
208.           diag(1)=d||' 95, since 10*25=(2x10)+(3x25)=20+75=95. Substitute n=10, m=25 in the formula n*m.';
209.           next=k13;
210.           GO TO now;
211.      k13:  PUT LIST('Since 25*10 is not the same value as 10*25, we can therefore conclude that the operation * is not');
212.           PUT LIST('_____on the set of whole numbers.');
```

ans(1)='COMMUTATIVE';

```

213.      wans(1)='ASSOCIATIVE';
214.      lw=1;
215.      cor='Good, we have shown * isn't commutative on W.';
216.      unrc='No, since 10*25 doesn't equal 25*10, we know that * isn't commutative on the set of whole numbers.';
217.      a='No, we have shown * isn't commutative. To show * isn't associative we would find values';
218.      b=' such that (a*b)*c and a*(b*c) were not equal.';
219.      diag(1)=a||b;
```



```

221.      next=k14;
222.      GO TO now;
223.      k14:  PUT LIST('To show that the operation $ is commutative, we must show that a$b=b$a for all whole numbers a and
b. ');
224.      PUT LIST('Since n$m=(n+m)/2, we know that 19$31=(19+31)/2=25, and 31$19=(31+19)/2=25. Does the fact that
19$31=31$19');
225.      PUT LIST('prove that $ is commutative? (Answer yes or no.)');
226.      ans(1)='NO';
227.      cor='that's right!';
228.      unrc='No!';
229.      next=k15;
230.      GO TO now;
231.      k15:  PUT LIST('The fact that 19$31=31$19 merely indicates that $ may be commutative--it doesn't prove it, because');
232.      PUT LIST('"$ is commutative on W" means that a$b=b$a for ALL values of a and b in the set W, not just for the
specific');
233.      PUT LIST('values 19 and 31. For this operation, however, a$b=(a+b)/2, and b$a=(b+a)/2. Since (a+b)/2=(b+a)/2 for
all');
234.      PUT LIST('whole numbers a and b (by the commutative property of addition), we thus know that a$b=b$a for all
whole');
235.      PUT LIST('numbers a and b; hence $ is commutative on W. Now look at the operation #, where n#m=(n*xm)-1. We
have');
236.      PUT LIST('already shown that 4#10=39. What is the value of 10#4?');
237.      ans(1)='39';
238.      wans(1)='-';
239.      lw=1;
240.      cor='Yes, 10#4=(10x4)-1=39.';
241.      unrc='No, if n#m=(n*xm)-1, then 10#4=(10x4)-1=39.';
242.      diag(1)=d||| '39: 10#4=(10x4)-1=40-1=39.';
243.      next=k16;
244.      GO TO now;
245.      k16:  PUT LIST('Thus a$b=b$a for one pair of numbers; to show # is commutative on W, we must still show that a#b=b#a');
246.      PUT LIST('for all elements of W, however. Again, this is not hard, since a#b=(axb)-1, and b#a=(bxa)-1. Thus');
247.      PUT LIST('a#b=(axb)-1=(bxa)-1=b#a, so # is commutative on W. ');
248.      PUT LIST('To see if the operation ** (previously defined) is commutative on W, we would want to see if a**b=_____
');
249.      PUT LIST('for all whole numbers a and b. ');
250.      ans(1)='B**A';
251.      cor='Fine, you are correct.';
252.      unrc='No, if ** is commutative on W, then a**b must equal b**a for all whole numbers a and b.';
253.      next=k17;
254.      GO TO now;
255.      k17:  PUT LIST('If n**m=n raised to the mth power, then 2**6=2x2x2x2x2=64. What is the numeric value of 6**2?');
256.      ans(1)='36';
257.      ans(2)='6x6';
258.      lr=2;
259.      cor='Right, 6**2=6x6=36.';
260.      unrc='Not quite: 6**2=6x6=36.';
261.      next=k18;
262.      GO TO now;
263.      k18:  PUT LIST('Thus 2**6 does not equal 6**2, so the operation ** is not commutative. Incidentally, 2**4=2x2x2x2=16,')
;
264.      PUT LIST('and 4**2=4x4=16, so it is possible that a**b=b**a for some, but not all, values of a and b. This is
why we');
265.      PUT LIST('can't show an operation is commutative by showing a**b=b**a for some specific values of a and b. ');
266.      PUT LIST('if subtraction is commutative, then it must be true that a-b=_____for all values of a and b. ');
267.      ans(1)='B-A';
268.      cor='Correct.';

```

```

269.      unrc='No, if subtraction is commutative, then a-b=b-a for all values of a and b.';
270.      next=k19;
271.      GO TO now;
272.  k19:  PUT LIST('This is not true, since 5-2 is not equal to 2-5, for instance; thus subtraction is not commutative');
273.      PUT LIST('on the set of whole numbers.');
```

PUT LIST('if subtraction is associative, then it must be true that for all whole numbers a,b, and c, (a-b)-c=a-(b-c).');

```

274.      PUT LIST('For instance, (9-6)-2, which is 3-2, or 1 must equal 9-(6-2), which is_____.(State the final value
only.)');
276.      ans(1)='5';
277.      wans(1)='-1';
278.      lw=1;
279.      cor='Yes, 9-(6-2)=9-(4)=5.';
280.      diag(1)='Your answer should not contain a minus sign. Actually, 9-(6-2)=9-4=5, so the answer is 5.';
281.      unrc='Not really. 9-(6-2)=9-4=5, so the answer is 5.';
282.      next=k20;
283.      GO TO now;
284.  k20:  END ;
```

—

```

55.  C3:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(30) VAR, wans(5) CHAR(20) VAR, cor CHAR(254) VAR, diag(5) CHAR(254) VAR, unrc CHAR(254) VAR;
57.      DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;
```

—

```

69.1     ans(1)='2-9';
69.2     ans(2)='-7';
69.3     lr=2;
69.4     b=' Thus W is not closed under subtraction.';
69.5     cor='Good, 2-9=-7 is not a whole number.'||b;
69.6     unrc='No, the number 2-9=-7 is not a whole number.'||b;
69.7     next=k21;
```

—

```

300.  k21: PUT LIST('if subtraction has an identity element, I, then I will have to satisfy the following two equations:');
301.      PUT LIST('a-I=a and I-a=a for all whole numbers a. Can you name an element that works in both equations? (if
there is)');
302.      PUT LIST('such an element, name it--if not, type "no".');
```

ans(1)='NO';  
wans(1)='0';  
wans(2)='ZERO';  
lw=2;

```

307.      b=' There is no element that always satisfies both equations.';
308.      diag(1)='No, zero works in the first equation only.'||b;
309.      diag(2)=diag(1);
310.      cor='You are correct,'||b;
311.      unrc='No,'||b;
312.      next=k22;
313.      GO TO now;
314.  k22:  next=k23;
315.      IF Index(reply,'0')>0 THEN GO TO next;
316.      IF Index(reply,'ZERO')>0 THEN GO TO next;
317.      PUT LIST('Is there a number, I, that always satisfies the equation a-I=a? If so, name it--if not, type "no."');
318.      b=' The number 0 always satisfies this equation.';
319.      diag(1)='Yes,'||b;
320.      diag(2)=diag(1);
320.5    lw=2;
321.      unrc='No, but'||b;
322.      cor='But there is such a number--the number 0.';
323.      GO TO now;
```

—

```

324.   k23:  PUT LIST('Thus there isn't an identity element for subtraction, because there is no element l such that ');
325.         PUT LIST('a-l=a and l-a=a for all whole numbers a. Since a-0=a for all values of a, we do say that 0 is a right
Identity');
326.         PUT LIST('for subtraction. (A right identity because it is written on the right side.) Incidentally, you know
that a-0=0');
327.         PUT LIST('because a-0 is defined as "the answer to the question ?+0=a", and you know that a+0=a for all whole
numbers a');
328.         PUT LIST('because 0 is the _____');
329.         ans(1)='IDENTITY';
330.         cor='Yes, 0 is the identity element for addition on W.';
331.         unrc='No, 0 is the identity element for addition on W.';
332.         next=k24;
333.         GO TO now;
334.   k24:  PUT LIST('End of lesson--you may logout. ');
335.         END ;

```

CAI UNIT 3

```

1. DECLARE Z1 ENTRY EXT;
1.1 CALL Z1;
2. DECLARE Z2 ENTRY EXT;
2.1 PUT LIST('Part 2: In case of malfunction, req 2 thru...');
2.2 PUT LIST('If you know that 5=10/2, which multiplication fact do you also know?');
2.3 CALL Z2;

-- 55. Z1: PROCEDURE ;
56. DECLARE ans(4) CHAR(30) VAR, cor CHAR(200) VAR, unrc CHAR(200) VAR;
57. DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

-- 69.1 PUT LIST('The operation of division has some properties of operations we have previously studied, but it also
falls');
69.2 PUT LIST('to have some properties of previous operations. If division is to be commutative on the set of whole
numbers');
69.3 PUT LIST('For instance, then for any whole numbers a and b it must be true that a/b=_____');
69.4 ans(1)='B/A';
69.5 cor='That''s right!';
69.6 unrc='No, if division is commutative, it must be true that a/b=b/a for all whole numbers a and b.';
69.7 next=k2;

-- 100. k2: PUT LIST('Thus if division is commutative, it must be true that 3/2=_____');
101. ans(1)='2/3';
102. cor='Good.';
103. unrc='No, if division is commutative, then 3/2 must equal 2/3';
104. next=k3;
105. GO TO now;
106. k3: PUT LIST('Thus division is not commutative, because 3/2 is not equal to 2/3. ');
107. PUT LIST('If for all whole numbers a,b, and c it is true that (a/b)/c = a/(b/c), then we say division is_____
');
-- 108. ans(1)='ASSOCIATIVE';
109. cor='Yes, you are correct.';
110. unrc='No, if (a/b)/c = a/(b/c) for all whole numbers a,b, and c, then division is said to be associative.';
111. next=k4;
112. GO TO now;
113. k4: PUT LIST('Clearly, (8/2)/1=4/1=4. What is the value of 8/(2/1)?(Type a numeral, not an expression.)');
114. ans(1)='4';
115. cor='Right, 8/(2/1)=8/2=4.';
116. unrc='No, 8/(2/1)=8/2=4.';
117. next=k5;
118. GO TO now;
119. k5: PUT LIST('Thus (8/2)/1 = 8/(2/1). Does this prove that division is associative?');
120. ans(1)='NO';
121. cor='Very good, one example doesn''t prove an operation is associative.';
122. unrc='No, a single example is never enough to show that an operation is associative.';
123. next=k6;
124. GO TO now;
125. k6: PUT LIST('As a matter of fact, division is NOT associative, because (8/4)/2=2/2=1, but 8/(4/2)=8/2=4; hence');
126. PUT LIST('(8/4)/2 is not equal to 8/(4/2). If division is distributive over addition, then two equations must be
true');
127. PUT LIST('a/(b+c)=a/b +a/c and (b+c)/a=b/a + c/a. Try both of these equations with the values a=6,b=2, and
c=1');
128. PUT LIST('Do these values work in both equations? Answer yes or no. ');
130. ans(1)='NO';
131. cor='Correct: the second equation works for these values, but the first equation doesn''t.';
132. unrc='No, the second equation works, but the first equation doesn''t: 6/(2+1)=6/3=2, but 6/2 + 6/1 = 3+6=9.';
133. next=k7;

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134. GO TO now;
135. k7: PUT LIST('Since  $6/(2+1)$  does not equal  $6/2 + 6/1$ , we know that there is no such thing as the distributive
property');
136. PUT LIST('of division over addition. The second equation, however, does hold true, and this property--');
137. PUT LIST('(b+c)/a = b/a + c/a --is called the right distributive property of division over addition. It is
called');
138. PUT LIST('the RIGHT distributive property, because the divisor, a, is written on the right. ');
140. PUT LIST('If there is an identity element, 1, for division, it will have to satisfy the following two equations:');
);
141. PUT LIST('1/a=a and a/1=a for all whole numbers a. Is there an element that satisfies both equations?(Yes or
no, )');
142. ans(1)='NO';
143. cor='Good, there is no element 1 such that 1/a=a for all whole numbers a.';
144. unrc='1 works in the second equation, but not in the first; there is no element that works in the first
equation.';
145. next=k8;
146. GO TO now;
147. k8: PUT LIST('Because 1 satisfies part of the requirements for being an identity element--namely, a/1=a for all
a--');
148. PUT LIST('we say that 1 is a right identity element for division. (A right identity element, because it is
written');
149. PUT LIST('on the right side of the division sign.) It is not, however, an identity element, because 1/a is not
equal');
150. PUT LIST('to a for all values of a. ');
151. PUT LIST('Now consider the actual definition of division: if p and d are whole numbers with d not zero, ');
152. PUT LIST('then p/d is the answer to the question ?xd=p. Thus 6/2, for instance, is the answer to the question
?x2=6. ');
153. PUT LIST('Similarly, 15/3 is the answer to the question_____. (Complete the sentence with a multiplication
sentence. ');
154. nrt=0;
155. lr=4;
156. ans(1)='?X3=15';
157. ans(2)='3X?=15';
158. ans(3)='15=?X3';
159. ans(4)='15=3X?';
160. cor='That's right, 15/3 answers the question ?X3=15.';
161. unrc='No, if p/d answers the question ?xd=p, then 15/3 should answer the question ?x3=15.';
162. next=k9;
163. GO TO now;
164. k9: PUT LIST('Which question does 35/7 answer?');
165. lr=4;
166. ans(1)='?X7=35';
167. ans(2)='7X?=35';
168. ans(3)='35=?X7';
169. ans(4)='35=?X7';
170. a='35/7 answers the question ?x7=35.';
171. cor='Good, '||a;
172. unrc='Not exactly: p/d answers the question ?xd=p, so '||a;
173. next=k10;
174. GO TO now;
175. k10: next=k12;
176. IF nrt=2 THEN GO TO next;
177. PUT LIST('Try another one: which question does 12/4 answer?');
178. ans(1)='?X4=12';
179. ans(2)='4X?=12';
180. ans(3)='12=?X4';
181. ans(4)='12=?X4';

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182.      lr=4;
183.      a='12/4 answers the question ?x4=12.';
184.      cor='Yes, '||a;
185.      unrc='No, if p/d answers the question ?xd=p, then '||a;
186.      next=k11;
187.      GO TO now;
188.      k11: next=k12;
189.           IF nrt=2 THEN GO TO next;
190.           PUT LIST('Which question does 20/5 answer?');
191.           lr=4;
192.           ans(1)='?X5=20';
193.           ans(2)='5X7=20';
194.           ans(3)='20=7X5';
195.           ans(4)='20=5X7';
196.           cor='Right.';
197.           unrc='Look, p/d answers the question ?Xd=p; 15/3 answers ?x3=15; 12/4 answers ?x4=12; then 20/5 answers ?x5=20';
198.           GO TO now;
199.      k12: END ;

- 55.      22: PROCEDURE ;
56.      DECLARE ans(4) CHAR(10) VAR, wans(5) CHAR(10) VAR, cor CHAR(200) VAR, diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;
57.      DECLARE a CHAR(100) VAR, b CHAR(100) VAR, c CHAR(100) VAR, d CHAR(50) VAR, e CHAR(50) VAR, reply CHAR(30) VAR;

- 69.1     lw=1;
69.2     lr=4;
69.3     ans(1)='2X5=10';
69.4     ans(2)='5X2=10';
69.5     ans(3)='10=2X5';
69.6     ans(4)='10=5X2';
69.7     wans(1)='?';
69.72    cor='Right.';
69.74    diag(1)='A multiplication fact shouldn't contain a question mark. The answer should be 2x5=10.';
69.76    unrc='No, if 10/5=2, then it must be true that 2x5=10.';
69.78    next=k13;

- 200.     k13: next=k14;
201.           IF nrt=1 THEN GO TO next;
202.           PUT LIST('If you know that 18/3=6, which multiplication fact do you also know?');
203.           lw=1;
204.           lr=4;
205.           ans(1)='3X6=18';
206.           ans(2)='6X3=18';
207.           ans(3)='18=3X6';
208.           ans(4)='18=6X3';
209.           cor='You are correct.';
210.           diag(1)='Multiplication FACTS don't contain question marks. In this case, if 18/3=6, then 3x6=18: 3x6=18 is the
answer.';
211.           unrc='No, 18/3 answers ?x3=18. Since 18/3=6, 6 must answer this question. Thus 6x3=18, so "6x3=18" is the
answer.';
212.           GO TO now;
213.           k14: PUT LIST('Zero has sometimes caused trouble in division. If we allow the expression 3/0, for instance, which
question?');
214.           PUT LIST('would 3/0 have to answer?');
215.           lr=4;
216.           ans(1)='?X0=3';
217.           ans(2)='0X?=3';
218.           ans(3)='?=0X3';

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219.      ans(4)='?3X0';
220.      cor='You are correct!';
221.      unrc='Not exactly!';
222.      next=k15;
223.      GO TO now;
224.      k15: PUT LIST('If 3/0 is defined at all, it must answer the question ?x0=3. But of course any number times zero is
zero,');
225.      PUT LIST('so there is no answer to this question. Thus division by 0 is undefined.');
```

226. PUT LIST('If 0/3 is defined at all, which question will it have to answer?');

```

227.      lr=4;
228.      ans(1)='?X3=0';
229.      ans(2)='?3X?0';
230.      ans(3)='?0=3X?';
231.      ans(4)='?0=?X3';
232.      a=' 0/3 must answer the question ?x3=0.';
233.      unrc='No,||a';
234.      cor='Fine,||a';
235.      next=k16;
236.      GO TO now;
237.      k16: PUT LIST('Is there an answer to this question?(Type yes or no.)');
```

238. ans(1)='NO';

239. wans(1)='YES';

```

240.      lw=1;
241.      cor='But there WOULD be an answer--zero answers this question, since 0x3=0. Thus 0/3 is defined: 0/3=0.';
242.      unrc=cor;
243.      diag(1)='Right, there would be an answer--zero answers this question, since 0x3=0. Thus 0/3 is defined: 0/3=0.';
244.      next=k17;
245.      GO TO now;
246.      k17: PUT LIST('We have not yet looked at the expression 0/0. If we allow the expression 0/0, which question would');
```

247. PUT LIST('it have to answer?');

```

248.      lr=4;
249.      ans(1)='?X0=0';
250.      ans(2)='?0X?0';
251.      ans(3)='?0=0X?';
252.      ans(4)='?0=?X0';
253.      a=' 0/0 must be the answer to the question ?x0=0.';
254.      cor='Yes,||a';
255.      unrc='Not exactly:||a';
256.      next=k18;
257.      GO TO now;
258.      k18: PUT LIST('One number that answers this question is____.(If you can think of a number that correctly answers
the');
259.      PUT LIST('question ?x0=0, then type that number-as a numeral, not a word. If not, type "impossible."');
```

260. lw=5;

261. ans(1)='IMPOSSIBLE';

262. wans(2)='E';

263. wans(1)='A';

264. wans(3)='I';

265. wans(4)='0';

266. wans(5)='U';

```

267.      b=' 865 answers this question, for instance, since 865x0=0.';
268.      cor='No, it isn't impossible--||b';
269.      unrc='You are absolutely correct!';
270.      diag(1)='I had wanted a numeral only, not a word answer. ||b';
270.1     diag(2)=diag(1);
270.2     diag(3)=diag(1);
270.4     diag(4)=diag(1);
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270.5      diag(5)=diag(1);
271.      next=k19;
272.      GO TO now;
273.      k19:  next=k20;
274.      tot=index(reply,'A')+index(reply,'E')+index(reply,'I')+index(reply,'O')+index(reply,'U');
275.      IF tot>0 THEN GO TO next;
276.      PUT LIST(reply||' x0=0. ');
276.5     a=reply;
277.      next=k21;
278.      GO TO next;
279.      k20:  a='865';
280.      k21:  nrt=0;
281.      PUT LIST('Now name another number that answers the question ?x0=0. ');
282.      lw=5;
283.      b='127 also answers this question, for instance, since 127x0=0. ';
284.      cor='it isn't impossible: '||b;
285.      diag(1)='I had wanted a numeral rather than a word answer. '||b;
286.      diag(2)=diag(1);
287.      diag(3)=diag(1);
288.      diag(4)=diag(1);
289.      diag(5)=diag(1);
290.      unrc='Right, ';
291.      next=k22;
292.      GO TO now;
293.      k22:  next=k23;
293.5     c=reply;
294.      tot=index(reply,'A')+index(reply,'E')+index(reply,'I')+index(reply,'O')+index(reply,'U');
295.      IF tot>0 THEN GO TO next;
295.5     next=sp;
296.      IF a=c THEN GO TO next;
297.      PUT LIST(c||' x0=0. ');
298.      next=k24;
299.      GO TO next;
300.      sp:  PUT LIST('However, I wanted a different number, not '||a||' again. '||b);
301.      k23:  c='127';
302.      k24:  PUT LIST('Thus both '||a||' and '||c||' answer the question ?x0=0, so there is no single answer to this
question. ');
303.      PUT LIST('In fact, any number will answer this question. That is why the expression 0/0 is undefined--it is
meaningless');
304.      PUT LIST('to talk about "the" answer to the question ?x0=0 when any number will answer this question. ');
305.      PUT LIST('End of lesson--you may logout now. ');
306.      END ;

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CAI UNIT 4

```

1.      DECLARE DEC1 ENTRY EXT;
1.1     CALL DEC1;
2.      DECLARE DEC2 ENTRY EXT;
2.1     PUT LIST('                                     Part 2: In case of malfunction, req
2 thru...');
2.2     PUT LIST('Another useful test is the following: a number is divisible by 3 if and only if the sum of its');
2.3     PUT LIST('digits is divisible by 3. For example, 171342 is divisible by 3, because the sum of its digits');
2.4     PUT LIST('is 1+7+1+3+4+2=18, and 18 is divisible by 3. The number 4123 is not divisible by 3, since the sum');
2.5     PUT LIST('of its digits is 4+1+2+3=10, and 10 is not divisible by 3. Is 25136 divisible by 3?');
2.6     CALL DEC2;
3.      DECLARE DEC3 ENTRY EXT;
3.1     PUT LIST('                                     Part 3: In case of malfunction, req
3 thru...');
3.2     PUT LIST('Another test that students often generalize is the rule for division by 6. Do you believe, for
Instance,');
3.3     PUT LIST('that a number is divisible by 10 if and only if it is divisible by 2 and by 5?');
3.4     CALL DEC3;
3.5     END ;

- 55.    DEC1:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(20) VAR, wans(3) CHAR(20) VAR, cor CHAR(200) VAR, dlag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
57.      DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

- 69.1    PUT LIST('If p and d are whole numbers with d not equal to zero, then p is divisible by d if the quotient p/d
is');
69.2    PUT LIST('a whole number. 6 is divisible by 2, for instance, since 6/2 is a whole number, but 6 is not divisible
);
69.3    PUT LIST('by 4, since 6/4 is not a whole number. ');
69.4    PUT LIST('it is often possible to tell whether p is divisible by d without actually performing the division.
You');
69.5    PUT LIST('probably already know the divisibility test for 2, for instance. Is 1432186 divisible by 2?');
69.6    ans(1)='YES';
69.7    cor='Right, 1432186 is divisible by 2.';
69.8    unrc='Actually, 1432186 is divisible by 2.';
69.82   next=k2;

- 100.   k2:   PUT LIST('In general, a number is divisible by 2 if and only if it's one's digit is divisible by 2. Thus');
101.     PUT LIST('1432186 is divisible by 2, because 6, the number in the one's position, is divisible by 2. ');
102.     PUT LIST('The test for divisibility by 5 is very similar: a number is divisible by 5 if and only if it's
one's');
103.     PUT LIST('digit is divisible by 5. For example 5 divides 1070 (since 5 divides 0) but 5 does not divide 259
(since)');
104.     PUT LIST('5 does not divide 9). Does 5 divide 55044?');
105.     ans(1)='NO';
106.     cor='Good, 5 won't divide 55044 because 5 won't divide 4.';
107.     unrc='Wrong: 5 won't divide the number in the one's position (4), so 5 won't divide the number 55044.';
108.     next=k3;
108.5   nrt=0;
109.     GO TO now;
110.   k3:   PUT LIST('Is 10536 divisible by 5?');
111.     cor='Fine, 10536 isn't divisible by 5.';
112.     qrt=nrt;
113.     nrt=0;
114.     unrc='You are incorrect: what is the number in the one's position of 10536?';
115.     next=k4;
116.     GO TO now;
117.   k4:   next=k5;

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118.      qrt=qrt+nrt;
119.      IF nrt>0 THEN GO TO next;
120.      ans(1)='6';
121.      cor='Yes, 6 is in the one's position of 10536, and 5 won't divide 6; hence 5 won't divide 10536.';
122.      unrc='No, 6 is in the one's position of 10536, and 5 won't divide 6; hence 5 won't divide 10536.';
123.      GO TO now;
124.      k5:  next=k6;
125.          IF qrt=2 THEN GO TO next;
126.          PUT LIST('Is 24130 divisible by 5?');
127.          ans(1)='YES';
128.          cor='You are right, 24130 is divisible by 5.';
129.          unrc='Wrong: 24130 is divisible by 5 because 0 (the number in the one's position) is divisible by 5.';
130.          GO TO now;
131.          k6:  nrt=0;
132.          PUT LIST('The test for divisibility by 4 is concerned not only with the last digit, but with the last two
digits');
133.          PUT LIST('of a number. The rule is this: a number is divisible by 4 if and only if the number named by the last
two');
134.          PUT LIST('digits is divisible by 4. For example, 21738 is not divisible by 4 because the number named by the
last');
135.          PUT LIST('two digits--38--is not divisible by 4. On the other hand, 21736 is divisible by 4, because 36--the
number');
136.          PUT LIST('named by the last two digits--is divisible by 4. Is 612854 divisible by 4?');
137.          ans(1)='NO';
138.          cor='Good: 4 doesn't divide 54, so 4 doesn't divide 612854.';
139.          unrc='You are not correct. What is the number named by the last two digits in 612854?';
140.          next=k7;
141.          GO TO now;
142.          k7:  next=k8;
143.              qrt=nrt;
144.              IF nrt>0 THEN GO TO next;
145.              ans(1)='854';
146.              lw=1;
147.              wans(1)='54';
148.              cor='No, the answer is 54, and since 4 won't divide 54, we know that 4 won't divide 612854.';
149.              unrc=cor;
150.              diag(1)='Yes, 54 is correct--and since 4 won't divide 54, we know that 4 won't divide 612854.';
151.              next=k8;
152.              GO TO now;
153.              k8:  PUT LIST('To see why this test works, consider the number 631218. This can be written as 631200+18');
154.                  PUT LIST('or (6312x100)+18. Obviously, 6312x100 is divisible by 4, so (6312x100 + 18) will be divisible by');
155.                  PUT LIST('4 if and only if 18 is divisible by 4. In general, a number edcba could be written as (edcx100 +ba).');
156.                  PUT LIST('Since edcx100 is always divisible by 4, the divisibility of (edcx100 + ba) depends only on the');
157.                  PUT LIST('divisibility of ba. Thus the divisibility test for 4 involves only the last two digits.');
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171. cor='No, 32 is the number named by the last 2 digits in 41732. 4 divides 32, so 4 divides 41732.';
172. unrc=cor;
173. dlag(1)='Good, 32 is correct; and since 4 divides 32, therefore 4 divides 41732.';
174. GO TO now;
175. k10: next=k11;
176. IF qrt=2 THEN GO TO next;
177. PUT LIST('Does 4 divide 218572?');
178. ans(1)='YES';
179. cor='Good, 4 divides 72, so 4 divides 218572.';
180. unrc='That is incorrect. 4 divides 72, so 4 divides 218572.';
181. GO TO now;
182. k11: END ;

- 55. DEC2: PROCEDURE ;
56. DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,dlag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
57. DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

- 69.1 ans(1)='NO';
69.2 cor='Very good, 2+5+1+3+6=17, and 17 isn't divisible by 3. Hence 25136 isn't divisible by 3.';
69.3 unrc='That is incorrect. What is the sum of the digits in 25136?';
69.4 next=k12;

- 200. k12: qrt=nrt;
201. next=k13;
202. IF nrt>0 THEN GO TO next;
203. ans(1)='17';
204. cor='Yes, 2+5+1+3+6=17. 17 isn't divisible by 3, so 25136 isn't divisible by 3.';
205. unrc='No, 2+5+1+3+6=17. 17, however, isn't divisible by 3, so 25136 isn't divisible by 3.';
206. GO TO now;
207. k13: PUT LIST('Is 14269 divisible by 3?');
208. ans(1)='NO';
209. cor='Fine, 1+4+2+6+9=22; 22 isn't divisible by 3, so 14269 isn't divisible by 3.';
210. unrc='Your answer is not right. What is the sum of the digits in 14269?';
211. nrt=0;
212. next=k14;
213. GO TO now;
214. k14: qrt=qrt+nrt;
215. next=k15;
216. IF nrt>0 THEN GO TO next;
217. ans(1)='22';
218. cor='22 is correct--and 3 won't divide 22, so 3 won't divide 14269.';
219. unrc='Nope, 1+4+2+6+9=22--and 3 won't divide 22, so 3 won't divide 14269.';
220. GO TO now;
221. k15: next=k16;
222. IF qrt=2 THEN GO TO next;
223. PUT LIST('Try another--is 82764 divisible by 3?');
224. ans(1)='YES';
225. cor='Right, 82764 is divisible by 3.';
226. unrc='That is incorrect. 8+2+7+6+4=27, and 3 divides 27, so 3 divides 82764.';
227. GO TO now;
228. k16: PUT LIST('The test for divisibility by 9 is analogous to the test for 3: a number is divisible by 9 if');
229. PUT LIST('and only if the sum of it's digits is divisible by 9. Is 250713 divisible by 9?');
230. ans(1)='YES';
231. cor='OK, 2+5+0+7+1+3=18, and 9 divides 18, so 9 divides 250713.';
232. unrc='That's incorrect. What is the sum of the digits in 250713?';
233. nrt=0;
234. next=k17;

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235.      GO TO now;
236. k17:  qrt=prt;
237.      next=k19;
238.      IF prt>0 THEN GO TO next;
239.      ans(1)='18';
240.      unrc='No, 2+5+0+7+1+3=18. 9 divides 18, so 9 divides 250713.';
241.      next=k18;
242.      GO TO now;
243. k18:  PUT LIST('Is 41682 divisible by 9?');
244.      ans(1)='NO';
245.      cor='Right: 4+1+6+8+2=21, and 9 won't divide 21, so 9 won't divide 41682.';
246.      unrc='Not quite: 4+1+6+8+2=21, and 9 won't divide 21, so 9 won't divide 41682.';
247.      next=k19;
248.      GO TO now;
249. k19:  PUT LIST('A final rule to be considered is the rule for divisibility by 6: a number is divisible by 6');
250.      PUT LIST('if and only if it is divisible by 2 and by 3. As an example, the number 35214 is divisible by 6');
251.      PUT LIST('because it is divisible by 2 (since its one's digit is divisible by 2) and it is divisible by 3');
252.      PUT LIST('(since the sum of its digits is divisible by 3). Is 2726 divisible by 6?');
253.      ans(1)='NO';
254.      cor='Good, 2726 isn't divisible by 6 because it isn't divisible by 3.';
255.      unrc='No, 2726 isn't divisible by 3, so it can't be divisible by 6.';
256.      prt=0;
257.      next=k20;
258.      GO TO now;
259. k20:  next=k21;
260.      IF prt>0 THEN GO TO next;
261.      PUT LIST('Try it again: Is 8223 divisible by 6?');
262.      unrc='You are incorrect: 8223 isn't divisible by 6 because it isn't divisible by 2.';
263.      cor='Correct, 8223 isn't divisible by 6 because it isn't divisible by 2.';
264.      GO TO now;
265. k21:  PUT LIST('Many students try to generalize these tests, but this must be done with caution. Consider');
266.      PUT LIST('the test for 3--the one that involves summing the digits. Will this test also work for divisibility');
267.      PUT LIST('by 7? That is, will a number be divisible by 7 if and only if the sum of its digits is divisible');
268.      PUT LIST('by 7? (Answer yes or no.)');
269.      ans(1)='NO';
270.      b=' 7 divides 14, for instance, but the sum of the digits is 1+4=5, and 7 won't divide 5.';
271.      cor='Correct--the rule won't work for 7.'||b;
272.      unrc='Wrong--the rule won't work for 7.'||b;
272.5    next=k22;
273.      GO TO now;
274. k22:  PUT LIST('In fact, the 3's rule won't work for any digits other than 3 and 9.');
```

END ;

```

- 55.  DEC3:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(20) VAR, wans(4) CHAR(20) VAR, cor CHAR(20) VAR, diag(4) CHAR(150) VAR, unrc CHAR(150) VAR;
57.      DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

- 69.1  ans(1)='YES';
69.2  cor='Fine, this test works--but there is an easier way to state it.';
69.3  unrc='Wrong! This test does work, although there is an easier way to state it.';
69.4  next=k23;

- 300. k23:  PUT LIST('A number that is divisible by 2 ends in a 0, 2, 4, 6, or 8, and a number that is divisible by 5');
301.      PUT LIST('ends in a 0 or a 5. Thus a number that is divisible by both 2 and 5 must end in the digit_____');
302.      ans(1)='0';
303.      ans(2)='ZERO';
304.      lr=2;

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305.      b=' a number that is divisible by both 2 and 5 ends in a 0.';
306.      cor='Yes, '||b;
307.      unrc='No, '||b;
308.      next=k24;
309.      GO TO now;
310. k24:  PUT LIST('Thus a number is divisible by 10 if and only if it ends in a 0. ');
311.      PUT LIST('Is it true that a number is divisible by 8 if and only if it is divisible by 2 and by 4? ');
312.      ans(1)='NO';
313.      cor='You are right, this test does not work.';
314.      unrc='Unfortunately, this test won't work.';
315.      next=k25;
316.      GO TO now;
317. k25:  PUT LIST('For example, 12 is divisible by 2 and by 4, but not by 8. Incidentally, the reason this test');
318.      PUT LIST('won't work for 2 and 4 is that 2 and 4 have a common factor. ');
319.      PUT LIST('From the following list, name a number that is divisible by 3: 46312, 46313, 46314, 46315, 46316 ');
320.      ans(1)='46314';
321.      lw=4;
322.      wans(1)='46312';
323.      wans(2)='46313';
324.      wans(3)='46315';
325.      wans(4)='46316';
326.      diag(1)='No, the sum of the digits in 46312 is 16, and 3 won't divide 16. The correct answer is 46314.';
327.      diag(2)='No, the sum of the digits in 46313 is 17, and 3 won't divide 17. The correct answer is 46314.';
328.      diag(3)='No, the sum of the digits in 46315 is 19, and 3 won't divide 19. The correct answer is 46314.';
329.      diag(4)='No, the sum of the digits in 46316 is 20, and 3 won't divide 20. The correct answer is 46314.';
330.      unrc='No, the only correct answer is 46314.';
331.      cor='Right, the sum of the digits in 46314 is 18, and 3 divides 18, so 3 divides 46314.';
332.      next=k26;
333.      GO TO now;
334. k26:  PUT LIST('From the same list, choose a number that is divisible by 4. ');
335.      lw=4;
336.      ans(1)='46316';
337.      wans(4)='46314';
338.      cor='Good, 4 divides 16, so 4 divides 46316.';
339.      unrc='No, the number named by the last two digits must be divisible by 4, and this is true only of 46312 and
46316.';
340.      diag(1)='Good, 4 divides 12, so 4 divides 46312.';
341.      diag(2)='No, 13 (the number named by the last 2 digits) isn't divisible by 4. The answer is 46312 or 46316.';
342.      diag(3)='No, 15 (the number named by the last 2 digits) isn't divisible by 4. The answer is 46312 or 46316.';
343.      diag(4)='No, 14 (the number named by the last 2 digits) isn't divisible by 4. The answer is 46312 or 46316.';
344.      next=k27;
345.      GO TO now;
346. k27:  PUT LIST('Name a number from the list that is divisible by 6. ');
347.      ans(1)='46314';
348.      lw=4;
349.      wans(4)='46316';
350.      cor='Yes, 46314 is divisible by both 2 and 3, so it is divisible by 6.';
351.      unrc='No, 46314 is the only listed number that is divisible by 2 and 3, so it is the only number divisible by
6.';
352.      diag(1)='No, 46312 isn't divisible by 3, so it isn't divisible by 6. The correct answer is 46314.';
353.      diag(2)='No, 46313 isn't divisible by 2 or 3, so it isn't divisible by 6. The correct answer is 46314.';
354.      diag(3)='No, 46315 isn't divisible by 2 or 3, so it isn't divisible by 6. The correct answer is 46314.';
355.      diag(4)='No, 16 isn't divisible by 3, so it isn't divisible by 6. The correct answer is 46314.';
356.      next=k28;
357.      GO TO now;
358. k28:  PUT LIST('End of lesson--you may logout. ');
359.      END ;

```

CAI UNIT 5



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1.      DECLARE PR1 ENTRY EXT;
1.1     CALL PR1;
2.      DECLARE PR2 ENTRY EXT;
2.1     PUT LIST('
part 2: in case of malfunction, req
2 thru...');
2.2     PUT LIST('What is the largest integer that is less than or equal to sqrt(140)?');
2.3     CALL PR2;
3.      DECLARE PR3 ENTRY EXT;
3.1     PUT LIST('
part 3: in case of malfunction, req
3 thru...');
3.2     PUT LIST('To see if 301 is prime, one should check the set of all primes that are less than or equal to
sqrt(301).');
3.3     PUT LIST('What is the largest prime number that is less than or equal to sqrt(301)?');
3.4     CALL PR3;
4.      END ;

- 55.    PR1:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(20) VAR, wans(3) CHAR(20) VAR, cor CHAR(200) VAR, diag(3) CHAR(200) VAR, unrc CHAR(200) VAR;
57.      DECLARE a CHAR(130) VAR, b CHAR(130) VAR, reply CHAR(30) VAR;

- 69.1    PUT LIST('You may remember that a prime number is a natural number that has exactly two factors. Alternately, we
could');
69.2    PUT LIST('say that a prime number is a natural number greater than 1 that has no divisors other than 1 and
itself. ');
69.3    PUT LIST('Is 21 a prime number?');
69.35   ans(1)='NO';
69.4    a=' 21 isn't prime, because it has 4 factors--1,3,7, and 21.';
69.5    cor='Right, '||a;
69.6    unrc='No, '||a;
69.7    next=k2;
69.8    GO TO now;

- 100.   k2:  PUT LIST('Is 33 a prime number?');
101.     a='33 isn't prime because 3 and 11 are divisors of 33; 33 is divisible by natural numbers other than 1 and 33.';
102.     cor='Good, '||a;
103.     unrc='Unfortunately not: '||a;
104.     next=k3;
105.     GO TO now;
106.   k3:  next=k6;
107.     IF nrt=2 THEN GO TO next;
108.     PUT LIST('Try again. Is 23 a prime number?');
109.     ans(1)='YES';
110.     a=' 23 is prime because it has no divisors other than 1 and 23.';
111.     cor='Very good, '||a;
112.     unrc='Wrong! '||a;
113.     next=k4;
114.     GO TO now;
115.   k4:  next=k6;
116.     IF nrt=2 THEN GO TO next;
117.     PUT LIST('Just one more: Is 77 a prime number?');
118.     ans(1)='NO';
119.     cor='Correct, 77 isn't a prime number.';
120.     unrc='No, 77 is not a prime number.';
121.     next=k5;
122.     GO TO now;
123.   k5:  PUT LIST('It is obvious that 77 is not prime because 77 is divisible by____.(Type one number.)');
124.     ans(1)='77';

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125.      wans(1)='11';
126.      wans(2)='1';
127.      wans(3)='7';
128.      b=' 77 fails to be prime because 7 and 11 are also divisors.';
129.      cor='77 is divisible by 77, but so what? Any number is divisible by itself.'||b;
129.5    lw=3;
130.      diag(2)='77 is divisible by 1, but so what? Any number is divisible by 1.'||b;
131.      diag(1)='Good, 77 fails to be prime because it is divisible by 11; it is also divisible by 7.';
132.      diag(3)='Good, 77 fails to be prime because it is divisible by 7; it is also divisible by 11.';
133.      next=k6;
134.      GO TO now;
135.      k6:  PUT LIST('For reasons that we will (hopefully) see later, we will be interested in square roots. In fact,
we''11');
136.      PUT LIST('be using the phrase "the square root of" so often that we will shorten it to "sqrt." Thus "the square
root');
137.      PUT LIST('of 9" will be written "sqrt(9)" in our terminology. What is the value of sqrt(9)?');
138.      ans(1)='3';
139.      cor='Yes, sqrt(9)=3.';
140.      nrt=0;
141.      unrc='No, sqrt(9)="the square root of 9"=3.';
142.      next=k7;
143.      GO TO now;
144.      k7:  next=k9;
145.          IF nrt=1 THEN GO TO next;
146.          PUT LIST('You see, sqrt(n) is the number which, when multiplied by itself, gives you n; that is, sqrt(n) is a
number');
147.          PUT LIST('whose square is n. In this case, sqrt(9)=3, since 3 is a number which, when multiplied by itself,
gives');
148.          PUT LIST('you 9. Similarly, sqrt(25)=5, since 5x5=25, and sqrt(100)=10, since 10x10=100. In general, then, ');
149.          PUT LIST('(sqrt(n))x(sqrt(n))=n. What is the numerical value of sqrt(64)?');
150.          ans(1)='8';
151.          cor='O.K., sqrt(64)=8.';
152.          unrc='Not quite: sqrt(64)=8, since 8x8=64. (8 is a number which, when multiplied by itself, gives you 64.)';
153.          next=k8;
154.          GO TO now;
155.          k8:  next=k9;
156.              IF nrt=1 THEN GO TO next;
157.              PUT LIST('Try one more: what is the numeric value of sqrt(121)?');
158.              ans(1)='11';
159.              cor='All right, sqrt(121)=11.';
160.              unrc='No, sqrt(121)=11. Sqrt(121) is a number whose square is 121; since 11x11=121, then, sqrt(121)=11.';
161.              GO TO now;
162.              k9:  PUT LIST('To decide whether 179 is prime or not, you are interested in discovering whether or not 179 has');
163.                  PUT LIST('any divisors other than 1 and 179. In this search for divisors, you would obviously not consider
numbers');
164.                  PUT LIST('that are larger than 179 as possible divisors of 179. In fact, you wouldn''t have to check any
numbers');
165.                  PUT LIST('that are larger than sqrt(179). By the way, what is the largest integer that is less than or equal to
sqrt(179)?');
166.                  ans(1)='13';
167.                  wans(1)='14';
168.                  lw=1;
169.                  cor='Very good, 13 is correct.';
170.                  diag(1)='You are close, but 14 is somewhat larger than sqrt(179). The correct answer is 13.';
171.                  unrc='No, 13 is the largest integer that is less than or equal to sqrt(179).';
172.                  next=k12;
173.                  GO TO now;

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174.   k12:   nrt=0;
175.         PUT LIST('Do you think you understand how to find the largest integer that is less than or equal to sqrt(n) for
a given n?');
176.         ans(1)='ES';
177.         cor='O.K., then try this one to make sure:';
178.         unrc='O.K., to find the largest integer that is less than or equal to 253, for instance, we would first make a
reasonable';
179.         next=k13;
180.         GO TO now;
181.   k13:   next=k14;
182.         IF nrt=1 THEN GO TO next;
183.         PUT LIST('guess. Since 100<253<400, we know that sqrt(100)<sqrt(253)<sqrt(400); therefore 10<sqrt(253)<20,
since');
184.         PUT LIST('10=sqrt(100) and 20=sqrt(400). The number we are looking for, then, is somewhere between 10 and 20. ');
185.         PUT LIST('We might guess 15, but 15x15=225, and 225<253, so 15<sqrt(253). Also, 16x16=256, and 253<256, so
sqrt(253)<16. ');
186.         PUT LIST('Thus 225<253<256, which means sqrt(225)<sqrt(253)<sqrt(256); hence 15<sqrt(253)<16. This means that
the');
187.         PUT LIST('largest integer that is less than or equal to sqrt(253) is 15. Always find the two integers that
sqrt(n) is');
188.         PUT LIST('between--your "answer" is the smaller of these two. ');
189.   k14:   END ;

- 55.   PR2:   PROCEDURE ;
56.         DECLARE ans(2) CHAR(20) VAR, wans(3) CHAR(20) VAR, cor CHAR(200) VAR, diag(3) CHAR(200) VAR, unrc CHAR(200) VAR;
57.         DECLARE a CHAR(130) VAR, b CHAR(130) VAR, reply CHAR(30) VAR;

- 69.1   ans(1)='11';
69.2   cor='You are right!';
69.3   unrc='No, the answer is 11. ';
69.4   next=k15;
69.5   GO TO now;

- 200.   k15:   next=k16;
201.         IF nrt=1 THEN GO TO next;
202.         PUT LIST('You see, 11x11=121, and 12x12=144. But sqrt(121)<sqrt(140)<sqrt(144), so 11<sqrt(140)<12. Thus 11 is');
203.         PUT LIST('the largest integer that is less than or equal to sqrt(140). ');
204.         PUT LIST('Try one more: what is the largest integer that is less than or equal to sqrt(203)? ');
205.         ans(1)='14';
206.         cor='Right, 14 is the answer. ';
207.         unrc='No, the answer is 14. Since 14x14<203<15x15, therefore 14<sqrt(203)<15. ';
208.         GO TO now;
209.   k16:   PUT LIST('Now let's consider why, in deciding if n is prime, we can limit our search for factors to numbers
that');
210.         PUT LIST('are less than or equal to sqrt(n). ');
211.         PUT LIST('If a and b are a pair of factors of n, (that is, axb=n), then at least one of the numbers a or b must
be less');
212.         PUT LIST('than or equal to sqrt(n). After all, if a>sqrt(n) and b>sqrt(n), then axb>(sqrt(n))x(sqrt(n)), which
means');
213.         PUT LIST('axb>n; hence a and b wouldn't be factors of n. ');
214.         PUT LIST('Thus for each pair of factors of n, at least one member of the pair will be less than or equal to
sqrt(n). ');
215.         PUT LIST('This means that if n has any factors at all--other than 1 and n--it will have at least one factor
that');
216.         PUT LIST('is less than or equal to sqrt(n). By this reasoning, then, if 179 has any factors, it must have a
factor');
217.         PUT LIST('that is less than or equal to _____. ');

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218.      ans(1)='SQRT(179)';
219.      ans(2)='SQRT179';
220.      lr=2;
221.      wans(1)='13';
222.      wans(2)='14';
223.      lw=3;
224.      cor='Very good.';
225.      unrc='No, if 179 has any factors, then it must have a factor that is less than or equal to sqrt(179).';
226.      diag(1)='Very good.';
227.      diag(2)='You are right, but you could be more exact.';
228.      wans(3)='ROOT';
229.      diag(3)='Please use the abbreviation sqrt in place of "square root." The answer is sqrt(179).';
230.      next=k17;
231.      GO TO now;
232.      k17:  PUT LIST('Since 13 is the largest integer that is less than or equal to sqrt(179), then if 179 has any factors');
233.          PUT LIST('at all, it must have a factor that is less than or equal to 13.');
```

has');  
234. PUT LIST('There is one fact that can cut our work even more: if n has a factor, f, (where f is not 1 or n) then n  
235. PUT LIST('a prime factor that is less than or equal to f. To see how this works, consider the number 1879, which  
236. PUT LIST('a factor, 297. 297 isn't prime, but it can be factored into two numbers, 9x33. Again, 9 and 33  
237. PUT LIST('but 9 can be factored into 3x3, and 3 is prime. Thus 1879 has a prime factor, 3.');

aren't prime.');

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238.          PUT LIST('In general, if f is a factor of n, then either f is prime (so we've found our prime factor), or f can  

239.          PUT LIST('factored into the product of two numbers, say g and h. If g or h is prime, we have again found our  

240.          PUT LIST('factor. If not, g and h can be factored, and so on. If the process is continued, a prime factor will  

241.          PUT LIST('675 has a factor, 135. It therefore has a prime factor that is less than or equal to 135. Name this  

242.          ans(1)='3';
243.          ans(1)='45';
244.          ans(2)='15';
245.          wans(1)='3';
246.          wans(2)='5';
247.          lr=2;
248.          lw=2;
249.          cor='No, this is a factor of 675, but not a prime factor. The only prime factors are 3 and 5.';
250.          diag(1)='Yes, 3 is a factor of 675. The other prime factor is 5.';
251.          diag(2)='Yes, 5 is a factor of 675. The other prime factor is 3.';
252.          unrc='No, the only prime factors of 675 are 3 and 5.';
253.          next=k18;
254.          GO TO now;
255.          k18:  PUT LIST('Thus if a number n has any factors at all, then it must have a prime factor that is less than or  

256.          PUT LIST('to sqrt(n). To check on whether n is prime, then, WE NEED ONLY CHECK THE PRIME NUMBERS THAT ARE LESS  

257.          PUT LIST('OR EQUAL TO SQRT(n).');


THAN');



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258.          PUT LIST('To see if 179 is prime, then, check the primes that are less than or equal to sqrt(179): that is,  

259.          PUT LIST('the numbers 2,3,5,7,11, and 13. If they are not factors of 179, then 179 HAS no factors (except 1 and  

260.          PUT LIST('Are any of these numbers actually factors of 179?');


179.));



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261.          ans(1)='NO';
262.          cor='Right--therefore 179 must be a prime number.';
263.          unrc='No, none of these numbers are factors of 179: thus 179 is a prime number.';

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264.         next=k19;
265.         GO TO now;
266.     k19:   END ;

- 55.     PR3:  PROCEDURE ;
56.         DECLARE ans(2) CHAR(20) VAR, wans(5) CHAR(20) VAR, cor CHAR(200) VAR, diag(5) CHAR(200) VAR, unrc CHAR(200) VAR;
57.         DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

- 69.1     ans(1)='17';
69.2     cor='Correct.';
69.3     unrc='No, 17x17<301<18x18, so 17<sqrt(301)<18; thus 17 is the largest integer that is < or = sqrt(301).';
69.4     next=k20;
69.5     GO TO now;

- 300.    k20:  PUT LIST('To see if 301 is prime, one would check the numbers 2,3,5,7,11,13,and 17 to see if any of these are');
302.    PUT LIST('factors of 301. If possible, name a number on this list that is a factor of 301. If there is none,
type "none."');
303.    ans(1)='17';
304.    wans(2)='NONE';
305.    wans(1)='7';
306.    cor='No, 17 isn't a factor of 301. The answer should be 7.';
307.    diag(1)='Very good, 301 is divisible by 7.';
308.    diag(2)='But there is a factor in that list--the number 7.';
308.5    unrc='There is a factor in that list--the number 7 is a factor of 301.';
309.    lw=2;
310.    next=k21;
311.    GO TO now;
312.    k21:  PUT LIST('To see if 253 is a prime, one would divide by the numbers 2,3,5,7,11, and 13. If possible, name a');
313.    PUT LIST('number on that list that is a factor of 253; otherwise, type "none."');
314.    ans(1)='11';
316.    cor='O.K., 11 is the only listed factor of 253.';
317.    unrc='No, 11 is a factor of 253, since 253=11x23.';
318.    next=k22;
319.    GO TO now;
320.    k22:  PUT LIST('List the set of all numbers that you should check to see if 91 is prime. Please list the numbers');
321.    PUT LIST('in order, and separate them by commas. ');
321.5    ans(1)='9';
322.    ans(2)='11';
323.    wans(1)='2,3,5,7';
324.    wans(2)='2,3,5,AND7';
325.    wans(3)='2,3,5AND7';
326.    lw=3;
327.    cor='No, the primes that are less than or equal to sqrt(91) are 2,3,5,and 7';
328.    lr=2;
329.    diag(1)='Fine, the primes less than or equal to sqrt(91) are 2,3,5, and 7.';
330.    diag(2)=diag(1);
331.    diag(3)=diag(1);
332.    unrc=cor;
333.    next=k23;
334.    GO TO now;
335.    k23:  PUT LIST('Try to divide 91 by each of the factors 2,3,5, and 7. Is 91 a prime number?');
335.1    ans(1)='NO';
335.2    cor='Correct, 7 divides 91, so 91 isn't prime.';
335.3    unrc='No, 7 divides 91, so 91 isn't prime.';
335.4    next=k24;
336.    GO TO now;
337.    k24:  PUT LIST('Try this method (dividing by all the primes less than or equal to sqrt(n)) to see if 139 is prime. ');

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338.      PUT LIST('Is it?');
339.      ans(1)='YES';
340.      b=' none of the primes 2,3,5,7,or 11 are divisors of 139: hence 139 is prime.';
341.      cor='Yes, '||b;
342.      unrc='No, '||b;
343.      next=k25;
344.      GO TO now;
345.      k25:  PUT LIST('Name the numbers from the following list that are prime numbers: 113, 115, 117. (if there are none,
type');
346.      PUT LIST('none. ');
347.      ans(1)='115';
347.5     lw=2;
348.      wans(1)='117';
349.      wans(2)='113';
350.      cor='No, 115 is divisible by 5: only 113 is prime.';
351.      diag(1)='No, 117 is divisible by 3; only 113 is prime.';
352.      diag(2)='Yes, 113 is prime, because it can not be divided by 2,3,5, or 7.';
353.      unrc='No, 113 is prime, but it is the only listed number that is prime.';
354.      next=k26;
355.      GO TO now;
356.      k26:  IF reply='113' THEN score=1; ELSE score=0;
357.      PUT LIST('Name the numbers from the following list that are prime numbers:295, 297, 299. If there are none, type
"none. ');
358.      ans(1)='NONE';
359.      b=' there are no primes on the list: 5 divides 295, 3 divides 297, and 13 divides 299.';
360.      cor='Correct, '||b;
370.      unrc='No, '||b;
371.      nrt=0;
372.      next=k27;
373.      GO TO now;
374.      k27:  next=k30;
375.      score=score+nrt;
376.      IF score=2 THEN GO TO next;
377.      PUT LIST('Name the numbers from the following list that are prime numbers: 287,289,291. If there are none, type
"none. ');
378.      ans(1)='287';
379.      ans(2)='291';
380.      wans(1)='289';
380.5     wans(2)='NONE';
381.      lr=2;
382.      lw=2;
383.      cor='No, 287 is divisible by 7, 289 is divisible by 17, and 291 is divisible by 3: there are no primes on the
list.';
383.5     diag(2)='You are right, there are no primes on the list: 7 divides 287, 17 divides 289, and 3 divides 291.';
384.      unrc=cor;
385.      diag(1)=cor;
386.      next=k28;
387.      GO TO now;
388.      k28:  next=k30;
389.      IF index(reply,'NONE')>0 THEN score=score+1;
390.      IF score=2 THEN GO TO next;
391.      PUT LIST('Only one number from this list is prime--name it: 119, 121, 123, 125, 127, 129');
392.      lw=5;
393.      ans(1)='119';
394.      wans(1)='121';
395.      wans(2)='123';
396.      wans(3)='125';

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```

397.      wans(4)='127';
398.      wans(5)='129';
399.      cor='No, 119 is divisible by 7.';
400.      diag(1)='No, 121 is divisible by 11.';
401.      diag(2)='No, 123 is divisible by 3.';
402.      diag(3)='No, 125 is divisible by 5.';
403.      diag(4)='Very good.';
404.      diag(5)='No, 129 is divisible by 3.';
404.5    unrc='Your answer is unrecognized--you may have typed the letter l in place of the numeral 1, for instance.';
405.      next=k29;
406.      GO TO now;
407.      k29:  next=sp;
408.          tot=index(reply,'119')+index(reply,'121')+index(reply,'123')+index(reply,'125')+index(reply,'129');
409.          IF tot>0 THEN GO TO next;
409.5    next=k30;
410.      IF index(reply,'127')>0 THEN GO TO next;
411.      sp:  PUT LIST('Using the same list again, try to name the prime number.');
```

412. lw=5;

413. next=k29s;

414. GO TO now;

415. k29s: IF index(reply,'127')=0 THEN PUT LIST('Actually, the prime number was 127.');

416. k30: PUT LIST('End of lesson--you may logout.');

417. END ;

CAI UNIT 6



```

1. DECLARE LCM1 ENTRY EXT;
1.2 CALL LCM1;
2. DECLARE LCM2 ENTRY EXT;
2.1 PUT LIST('
thru...');
2.2 PUT LIST('For example, consider LCM(12,63). 12=2x2x3, and 63=3x3x7. Thus LCM(12,63) must contain the factor 2x2
(or');
2.3 PUT LIST('It wouldn't be a multiple of 12), 3x3 (or it wouldn't be a multiple of 63), and 7 (or it wouldn't
be a');
2.4 PUT LIST('multiple of 63). Hence LCM(12,63)=_____. Again, just type the final answer, not the factors. ');
2.5 CALL LCM2;
3. DECLARE LCM3 ENTRY EXT;
3.1 PUT LIST('
thru...');
3.2 PUT LIST('O.K., so the least common denominator is 36. Now complete the addition: 5/12 + 7/18 =_____. ');
3.3 CALL LCM3;
4. END ;

-- LCM1: PROCEDURE ;
55. DECLARE ans(2) CHAR(20) VAR, wans(2) CHAR(20) VAR, cor CHAR(150) VAR, diag(2) CHAR(150) VAR, unrc CHAR(150) VAR;
56. DECLARE a CHAR(150) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;
57.

-- 69.1 PUT LIST('The least common multiple of a and b--abbreviated LCM(a,b)--is the smallest non-zero number that is a
multiple');
69.2 PUT LIST('of both a and b. The set of non-zero multiples of 12, for instance, contains the numbers 12,24,36,48,60
,72,...');
69.3 PUT LIST('and the set of non-zero multiples of 18 contains 18,36,54,72,90,... From this, it should be clear
that');
69.4 PUT LIST('LCM(12,18)=____. ');
69.5 ans(1)='36';
69.6 cor='Fine, LCM(12,18)=36. ';
69.7 unrc='No, the smallest number that is a non-zero multiple of both 12 and 18 is 36. ';
69.8 next=k2;

--100. k2: PUT LIST('We could, then, find LCM(a,b) in this way: let A=the set of non-zero multiples of a, and let B=the set
of');
101. PUT LIST('non-zero multiples of b. Then LCM(a,b) is the smallest member of the _____ of A and B. ');
102. ans(1)='INTERSECTION';
103. cor='Correct. ';
104. a='Not quite: LCM(a,b) is the smallest member that is in both A and B. ';
105. unrc='all' so it's the smallest member of the intersection of A and B. ';
106. next=k3;
107. GO TO now;
108. k3: PUT LIST('Try another one. If the non-zero multiples of 30 are 30,60,90,120,150,180,210,240,270,... and the
non-zero');
109. PUT LIST('multiples of 24 are 24,48,72,96,120,144,168,192,216,240,264,..., then what is LCM(30,24)? ');
110. ans(1)='120';
111. cor='Yes, LCM(30,24)=120. ';
112. unrc='No, the smallest element that is in both sets of multiples is 120: thus LCM(30,24)=120. ';
113. next=k4;
114. GO TO now;
115. k4: PUT LIST('Unfortunately, this is not a very practical way to find LCM(a,b), because you may have to calculate a
lot');
116. PUT LIST('of multiples before you find one that is in both sets. Another--easier--way to find LCM(a,b) is to
factor');
117. PUT LIST('a and b into a product of primes, and then TAKE THE PRODUCT OF THE HIGHEST POWERS OF EACH OF THE
DIFFERENT');

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Part 2: In case of malfunction, req 2

Part 3: In case of malfunction, req 3

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118.          PUT LIST('FACTORS THAT OCCUR IN THE PRIME FACTORIZATION OF EITHER NUMBER.');
```

factorization');  
119. PUT LIST('As a start, let's again find LCM(30,24). The prime factorization of 24 is 2x2x2x3, and the prime  
120. PUT LIST('of 30 is\_\_\_\_. List the factors in order, smallest to largest, with the letter x as a times sign  
between factors.');

```

121.          ans(1)='2X2';
122.          ans(2)='5X5';
123.          wans(1)='2X3X5';
124.          cor='Not exactly. 30, written as a product of primes, is 2x3x5.';
125.          diag(1)='Good, 30=2x3x5.';
126.          lr=2;
127.          lw=1;
128.          unrc=cor;
129.          next=k5;
130.          GO TO now;
131.          PUT LIST('All right, if 24=2x2x2x3, and 30=2x3x5, then "the different factors that occur in the prime factorizati  

on of');
```

132. PUT LIST('either number" are 2,3,and 5. The highest power of 2 in either number is 2x2x2 (which appears in 24),  
and the');

133. PUT LIST('highest power of 3 is simply 3; similarly, the highest power of 5 in either number is 5 to the first  
power.');

```

134.          PUT LIST('Thus LCM(30,24)=2x2x2x3x5=120.');
```

135. k5: PUT LIST('Try one on your own: what is LCM(90,100)? Please don't state the number in factored form--just the  
final');

```

136.          PUT LIST('answer, which will be a single number.');
```

```

137.          ans(1)='9000';
138.          wans(1)='X';
138.5        wans(2)='900';
139.          lw=2;
140.          a='Please state the final answer, not a list of factors. The answer is';
141.          diag(1)=all' 900.';
142.          cor='You are incorrect. Let's analyze the way to get the correct answer.';
143.          unrc=cor;
144.          diag(2)='Good, LCM(90,100)=900.';
145.          next=k6;
146.          GO TO now;
147.          k6:  next=det;
148.          IF INDEX(reply,'9000')>0 THEN GO TO next;
149.          next=k8;
150.          IF INDEX(reply,'900')>0 THEN GO TO next;
151.          det: PUT LIST('If 90=2x3x3x5, and 100=2x2x5x5, then the different prime factors occurring in either number are 2,3,and  

5.');
```

```

152.          PUT LIST('The highest power of 2 that occurs in either number is 2 to the_____power.');
```

```

153.          ans(1)='SECOND';
154.          ans(2)='2ND';
155.          lr=2;
156.          cor='Yes, 2 to the second power is the highest power of 2 in either 90 or 100.';
157.          unrc='No, 2x2, or 2 to the second power, is the highest power of 2 in either 90 or 100.';
158.          next=k7;
159.          GO TO now;
160.          k7:  PUT LIST('Similarly, the highest power of 3 appearing in either number is 3x3, and the highest power of 5 is  

5x5.');
```

161. PUT LIST('Thus "the product of the highest powers of each of the different factors that occur in either number"  
will be');

```

162.          PUT LIST('2x2x3x3x5x5=900. Hence LCM(90,100)=900.');
```

163. k8: PUT LIST('Now find LCM(8,30).');

```

164.          ans(1)='120';
```

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165.      wans(1)='X';
166.      lw=1;
167.      diag(1)=a||'120: 2x2x2x3x5=120.';
168.      cor='Fine, 2x2x2x3x5=120 is the LCM(8,30).';
169.      unrc='Not exactly: 8=2x2x2, and 30=2x3x5, so again we are dealing with the factors 2,3, and 5. Now the highest
power of';
170.      nrt=0;
171.      next=k9;
172.      GO TO now;
172.5   k9:  next=k10;
173.      IF nrt=1 THEN GO TO next;
173.5     IF Index(reply,'X')>0 THEN GO TO next;
174.      PUT LIST('2 in either number is 2x2x2; the highest power of 3 is 3 to the first power, and the highest power of
5 is');
175.      PUT LIST('5 to the first power. Then LCM(8,30)=_____. (Again, just type the final answer, not all the
factors.)');
176.      alw=1;
177.      unrc='No, LCM(8,30)=2x2x2x3x5=120--the product of the highest powers of the different factors present in either
number.';
178.      GO TO now;
179.      k10: PUT LIST('You see, any multiple of n and m must contain the highest power of each of the prime factors appearing');
180.      PUT LIST('in n (or else it wouldn't be a multiple of n) and the highest power of each of the prime factors
appearing');
181.      PUT LIST('in m (or else it wouldn't be a multiple of m). The smallest multiple of both n and m should be the
number that');
182.      PUT LIST('contains only these factors--hence the rule about taking the product of the highest power of each of
the prime');
183.      PUT LIST('factors that occur in either number.');
```

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184.      END;

-- 55.   LCM2:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
57.      DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;

-- 69.1   ans(1)='252';
69.2   a='Please give the final answer,';
69.3   wans(1)='X';
69.4   lw=1;
69.5   cor='Right, LCM(12,63)=252.';
69.6   unrc='No, LCM(12,63)=2x2x3x3x7=252.';
69.7   diag(1)=a||' 252: LCM(12,63)=2x2x3x3x7=252.';
69.8   next=k11;
69.9   GO TO now;

-- 200.   k11:  next=k12;
201.      IF nrt=1 THEN GO TO next;
202.      PUT LIST('Try one more by this method: find LCM(30,100).');
```

```

203.      ans(1)='3000';
204.      wans(2)='300';
205.      lw=2;
206.      diag(1)=a||' 300: 2x2x3x5x5=300.';
207.      diag(2)='Good, LCM(30,100)=2x2x3x5x5=300.';
208.      cor='No, LCM(30,100)=2x2x3x5x5=300.';
209.      unrc=cor;
210.      GO TO now;
211.      k12:  PUT LIST('There is another method of finding the LCM(a,b) that works when you already know the GCD(a,b)--the ');
```

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212. PUT LIST('greatest common divisor of a and b. The formula  $LCM(a,b)=(axb)/GCD(a,b)$  can then be used. For
example,');
213. PUT LIST('GCD(12,63)=3. By the formula, then,  $LCM(12,63)=(12x63)/GCD(12,63) = (12x63)/3 = 4x63=252$ . (What you are
really');
214. PUT LIST('doing when you divide by GCD(a,b) is removing all the factors that belong to both a and b--hence you
get rid of');
215. PUT LIST('the unnecessary factors in the product  $axb$ .)');
216. PUT LIST('Use this method to find  $LCM(72,120)$ , given that  $GCD(72,120)=24$ . Again, just state the final answer.');
```

ans(1)='360';  
wans(1)='/';  
wans(2)='X';  
lw=2;  
cor='Good,  $LCM(72,120)=(72x120)/24 =360$ .';  
diag(1)=a||' 360-- $(72x120)/24=360$ .';  
diag(2)=diag(1);  
unrc='Not quite:  $LCM(72,120)=(72x120)/GCD(72,120)=(72x120)/24 =360$ .';  
next=k13;  
GO TO now;

k13: PUT LIST('Use the same method to find  $LCM(90,300)$  given that  $GCD(90,300)=30$ .');

ans(1)='9000';  
lw=3;  
diag(1)=a||' 900-- $(90x300)/30=900$ .';  
wans(3)='900';  
unrc='not exactly-- $LCM(90,300)=(90x300)/GCD(90,300)=(90x300)/30 =900$ .';  
cor=unrc;  
diag(2)=diag(1);  
diag(3)='Correct,  $LCM(90,300)=900$ .';  
next=k14;  
GO TO now;

k14: PUT LIST('Find  $LCM(294,252)$  given that  $GCD(294,252)=42$ .');

ans(1)='1764';  
lw=2;  
cor='Yes,  $LCM(294,252)=(294x252)/42=1764$ .';  
unrc='No,  $LCM(294,252)=(294x252)/GCD(294,252) = (294x252)/42 =1764$ .';  
diag(1)=a||' 1764-- $(294x252)/42=1764$ .';  
diag(2)=diag(1);  
next=k15;  
nrt=0;  
GO TO now;

k15: next=k16;  
IF nrt=1 THEN GO TO next;  
PUT LIST('Just one more like this: find  $LCM(75,90)$ , given that  $GCD(75,90)=15$ .');

ans(1)='450';  
lw=2;  
cor='Fine,  $450=LCM(75,90)$ .';  
unrc='Not quite:  $LCM(75,90)=(75x90)/GCD(75,90) = (75x90)/15 =450$ .';  
diag(1)=a||' 450-- $(75x90)/15 =450$ .';  
diag(2)=diag(1);  
GO TO now;

k16: PUT LIST('The method used in the last few problems is useful when the numbers involved are extremely large--so
large');  
PUT LIST('that you would not want to factor the numbers into a product of primes. Instead, you can find  $GCD(a,b)$ 
by');  
PUT LIST('using Euclid's algorithm, and then use the formula  $LCM(a,b)=axb/GCD(a,b)$  to find the least common
multiple.');

PUT LIST('By now you may be wondering why anybody would ever want to find  $LCM(a,b)$ . Actually, least common
multiples are');

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265.      PUT LIST('useful in adding (or subtracting) a certain class of numbers; least common multiples are useful in
adding and');
266.      PUT LIST('subtracting_____');
267.      ans(1)='FRACTION';
268.      cor='right!';
269.      unrc='I didn't recognize your answer. Actually, least common multiples are used in adding fractions.';
270.      next=k17;
271.      GO TO now;
272.      k17:  PUT LIST('For instance, to add 5/12 + 7/18, you are interested in finding the least common denominator--that
is,');
273.      PUT LIST('the least common multiple of 12 and 18. But 12=2x2x3, and 18=2x3x3, so LCM(12,18)=_____.Type the final
answer.');
```

```

274.      ans(1)='36';
275.      wans(1)='X';
276.      lw=1;
277.      cor='Fine, LCM(12,18)=36.';
278.      diag(1)=a||' 36--2x2x3x3=36.';
279.      unrc='Actually, the answer is 36: 2x2x3x3=36.';
280.      next=k18;
281.      GO TO now;
282.      k18:  END ;
```

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55.      LCM3:  PROCEDURE ;
56.      DECLARE ans(2) CHAR(20) VAR,wans(3) CHAR(20) VAR, cor CHAR(200) VAR,diag(3) CHAR(150) VAR, unrc CHAR(150) VAR;
57.      DECLARE a CHAR(120) VAR, b CHAR(120) VAR, reply CHAR(30) VAR;
```

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-
69.1     ans(1)='29/36';
69.2     cor='Good, 29/36 is correct.';
69.3     unrc='No, 5/12 + 7/18 = 15/36 + 14/36 = 29/36.';
69.4     next=k19;
```

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-
300.      k19:  next=kk19;
301.      IF nrt=1 THEN GO TO next;
302.      PUT LIST('Remember that to add fractions that have the same denominator, you simply add the numerators and
keep');
303.      PUT LIST('the denominator the same. For instance, 5/8 + 1/8 = (5+1)/8 = 6/8. If the denominators are not
equal,');
304.      PUT LIST('however, it is necessary to first change to a common denominator, and the most efficient common
denominator');
```

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306.      PUT LIST('is the LCM of the denominators of the fractions. Remember that it is legal to multiply both numerator
and');
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307.      PUT LIST('denominator of a fraction by the same non-zero quantity: 2/5 = (2x6)/(5x6) = 12/30.');
```

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308.      kk19:  PUT LIST('The lowest common denominator that could be used to add 3/16 + 1/20 is_____');
309.      ans(1)='80';
310.      cor='Correct, LCM(16,20)=80.';
311.      unrc='Not exactly. The lowest common denominator is LCM(16,20), which is 2x2x2x2x5=80.';
312.      score1=nrt;
313.      nrt=0;
314.      next=k20;
315.      GO TO now;
316.      k20:  PUT LIST('Thus 3/16 + 1/20 =_____. (Final answer only, please.)');
```

```

317.      ans(1)='19/80';
318.      cor='Fine, 3/16 + 1/20 = 19/80.';
319.      score2=nrt;
320.      nrt=0;
321.      next=k21;
322.      unrc='No, 3/16 + 1/20 = 15/80 + 4/80 = 19/80.';
-

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```

323.      GO TO now;
324.      k21:  score1=score1+nrt;
325.          next=k22;
326.          IF nrt=1 THEN GO TO next;
327.          PUT LIST('Multiplying the numerator and denominator of a fraction by the same non-zero quantity doesn't change
the');
328.          PUT LIST('value of the fraction, because this multiplication is really multiplication by n/n, and n/n=1 for all
n not');
329.          PUT LIST('equal to zero. In the previous problem, then, we wanted to use 80 as a denominator, so we could have
looked');
330.          PUT LIST('at the problem as follows:  $3/16 + 1/20 = 7/80 + 7/80$ . To get 80 from the denominator 16, it was
necessary to');
331.          PUT LIST('multiply the denominator by 5; hence we must also multiply the numerator by 5, so  $3/16 = (3 \times 5)/(16 \times 5) = 15/80$ .');
332.          PUT LIST('Similarly, we must multiply numerator and denominator of  $1/20$  by 4:  $1/20 = (1 \times 4)/(20 \times 4) = 4/80$ . Hence
the');
333.          PUT LIST('addition problem is  $3/16 + 1/20 = 15/80 + 4/80 = 19/80$ .');
334.      k22:  PUT LIST('The lowest common denominator that could be used to add  $1/36 + 1/45$  is_____');
335.          ans(1)='180';
336.          cor='Fine, LCM(36,45)=180.';
337.          unrc='No, LCM(36,45)=2x2x3x3x5=180.';
338.          next=k23;
339.          nrt=0;
340.          GO TO now;
341.      k23:  score2=score2+nrt;
342.          next=k30;
343.          IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
344.          PUT LIST('Thus  $1/36 + 1/45 =$ _____.(Final answer only, please.)');
345.          ans(1)='9/180';
346.          ans(2)='1/20';
347.          lr=2;
348.          nrt=0;
349.          cor='Very good,  $1/36 + 1/45 = 9/180 = 1/20$ .';
350.          unrc='No,  $1/36 + 1/45 = 5/180 + 4/180 = (5+4)/180 = 9/180 = 1/20$ .';
351.          next=k24;
352.          GO TO now;
353.      k24:  next=k30;
354.          score1=score1+nrt;
355.          IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
356.          PUT LIST('Try another one: what is the smallest common denominator that one could use to add  $7/60 + 8/75$ ?');
357.          ans(1)='300';
358.          cor='Right, LCM(60,75)=300.';
359.          unrc='I'm afraid not. LCM(60,75)=2x2x3x5x5=300.';
360.          nrt=0;
361.          next=k25;
362.          GO TO now;
363.      k25:  next=k30;
364.          score2=score2+nrt;
365.          IF score1=(score1-1)*(score2-1)>0 THEN GO TO next;
366.          PUT LIST('Then  $7/60 + 8/75 =$ _____.(Final answer, please.)');
367.          ans(1)='67/300';
368.          cor='Good, the answer is 67/300.';
369.          unrc='Not quite:  $7/60 + 8/75 = 35/300 + 32/300 = 67/300$ .';
370.          next=k26;
371.          nrt=0;
372.          GO TO now;
373.      k26:  next=k30;

```

```

374. score1=score1+nrt;
375. IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
376. PUT LIST('Try some new numbers. Have the least common denominator that could be used to add 2/105 + 9/70.');
```

k27:

```

377. ans(1)='210';
378. cor='O.K., LCM(105,70)=210.';
379. unrc='No, LCM(105,70)=2x3x5x7=210.';
380. next=k27;
381. nrt=0;
382. GO TO now;
383. score1=score1+nrt;
384. next=k30;
385. IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
386. PUT LIST('Hence 2/105 + 9/70 = _____.');
```

k28:

```

387. ans(1)='31/210';
388. cor='Good, 31/210 is right.';
389. unrc='No, 2/105 + 9/70 = 4/210 + 27/210 = 31/210.';
390. next=k28;
391. nrt=0;
392. GO TO now;
393. score1=score1+nrt;
394. next=k30;
395. IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
396. PUT LIST('The least common multiple of 70 and 28 is _____.');
```

k29:

```

397. ans(1)='140';
398. cor='Good, LCM(70,28)=140. Hence the least common denominator used in adding 3/70 + 5/28 is 140.';
399. unrc='No, LCM(70,28)=2x2x5x7=140; thus the least common denominator used in adding 3/70 + 5/28 is 140.';
400. next=k29;
401. nrt=0;
402. GO TO now;
403. score2=score2+nrt;
404. next=k30;
405. IF score1*(score1-1)*(score2-1)>0 THEN GO TO next;
406. PUT LIST('Just one more: 3/70 + 5/28 = _____. (Again, just the final answer.));
407. ans(1)='31/140';
408. unrc='Not quite: 3/70 + 5/28 = 6/140 + 25/140 = 31/140.';
409. cor='Good, 31/140 is correct.';
410. GO TO now;
411. k30: PUT LIST('End of lesson--you may logout.');
```

k12:

```

412. END ;
```